Planning mathematical inquiry

To plan authentic mathematical inquiry, teachers must consider the following questions.

- **What do we want students to learn?**
- **What do teachers need to learn about this topic?**
- **How best will students learn?**
- **How will we know what students have learned?**

**What do we want students to learn?**

The mathematics scope and sequence framework identifies the expectations considered essential in the Primary Years Programme (PYP). These expectations are arranged in five interwoven strands of knowledge. In **number** and **pattern and function**, students inquire into our number system, and its operations, patterns and functions. This is where students become fluent users of the language of arithmetic, as they learn to understand its meanings, symbols and conventions. These two strands are best taught as stand-alone mathematical topics, although aspects can be successfully included in the units of inquiry.

The remaining strands, **data handling**, **measurement** and **shape and space**, are the areas of mathematics that other disciplines use to research, describe, represent and understand aspects of their domain. Mathematics provides the models, systems and processes for handling data, making and comparing measurements, and solving spatial problems. As a discipline it has no interest in, or ability to determine, the content of charts, graphs, tables, shapes or measurements. The content must come from the theme or area under study. Consequently, topics in these three strands are best studied in authentic contexts such as the transdisciplinary units of inquiry.

**What do teachers need to learn about this topic?**

The PYP views mathematics not as a fixed body of knowledge to be transmitted, but as a way of thinking and a language for understanding meaning. To study mathematics is to inquire into this language and to learn to think in this way. We believe teachers can use the eight key concepts and related questions (Figures 5 and 6 in *Making the PYP happen*) to guide their own inquiry. By engaging in inquiry themselves, teachers will not only achieve a deeper understanding of the mathematical issues involved, but will also provide a model for their students of the teacher as a learner.

Sample questions for each expectation and for each strand have been provided. Each of the questions may be linked to one or more concepts by the teacher. Some of the sample questions have been linked to an appropriate concept as examples.

Personal knowledge of the subject matter is of key importance. What teachers understand themselves will shape how well they select from activities and texts available, and how effectively they teach. The teacher’s personal interest in, and development of, the discipline should be maintained through regular professional development, reading professional journals and regular contact with colleagues who share their commitment to teaching mathematics through inquiry.
How best will students learn?

Cognitive psychologists have described the stages through which children learn mathematics. It is useful to identify these stages when designing developmentally appropriate learning experiences. The scope and sequence document uses and identifies these stages as follows.

- **Constructing meaning**
  Teachers plan activities, through which students construct meaning from direct experience, by using manipulatives and conversation.

- **Transferring meaning into signs and symbols**
  Teachers connect the notation system with the concrete objects and associated mathematical processes. The teacher provides the symbols for the students. Students begin to describe their understanding using signs and symbols.

- **Understanding and applying**
  Teachers plan authentic activities in which students independently select and use appropriate symbolic notation to process and record their thinking.

As they work through these stages, students use certain processes of mathematical reasoning.

- They use patterns and relationships to analyse the problem situations upon which they are working.
- They make and evaluate their own and each other’s ideas.
- They use models, facts, properties and relationships to explain their thinking.
- They justify their answers and the processes by which they arrive at solutions.

In this way, students validate the meaning they construct from their experiences with mathematical situations. By explaining their ideas, theories and results, both orally and in writing, they invite constructive feedback and also lay out alternative models of thinking for the class. Consequently, all benefit from this interactive process.

What does a PYP mathematics classroom look like?

In a PYP classroom, mathematics is a vital and engaging part of students’ lives. Students in the classroom are very active with an underlying sense of organization and cooperation. Teachers and students are asking questions of each other, trying out and demonstrating ideas in small and large groups, using the language of mathematics to describe their thinking, generating data to look for patterns, and making conjectures.

The value of open questions cannot be over-emphasized. When asked a closed question, students quickly learn that the accuracy of their answer is important. This may develop into an anxiety over getting the answers “wrong”. Open questions help students to develop their understanding and allow them an opportunity to think about their work and to verbalize their learning. Such probing questions quickly reveal those who have understood and those who have guessed. From the earliest stages, correct and appropriate mathematical language should be used. Consistent use will enable students to remember and use relevant vocabulary with confidence.

The use of appropriate technology in mathematics influences and enhances student learning. The computer area is well stocked with innovative software that encourages the application of mathematics skills and problem solving.

A wide variety of materials is available to all, ranging from student collections of keys, foreign money and seashells to store-bought cubes and counters, pattern blocks and geoboards. Everyday tools, such as measuring jugs, calculators and even cereal boxes, are in use. It is important that students are given the opportunity to select from a range of materials to enable them to generalize their understanding.
The constant use of appropriate *manipulatives* is an essential aspect of PYP mathematics. A variety of different materials should be used for each strand and expectation, and should continue to be used all the way through the programme.

The number of mathematics resources available is impressive. There are lists, tables and charts on display, showing written and numerical data, about which relevant questions are asked and answered. Colourful and thought-provoking posters and children’s work cover the walls. On the bookshelves are many resource books for the teacher and students, including a wide variety of textbooks, mathematical dictionaries and encyclopedias as well as children’s literature that focuses on mathematical ideas. The general supplies area has a variety of paper for recording mathematical ideas: different sized squared paper, dot paper and scribble paper. There may be a video area where high-quality video tapes show how mathematics is used outside the classroom. It is clear where things belong and how they are used.

The mathematics classroom does not work on its own. The students may visit younger children to help them with their investigations, or work with older children to travel about town in small groups on a treasure hunt, asking and answering questions about dates, times, distance, prices and more. Other teachers and parents come to share their interests and expertise. The school nurse and school administrators are involved in providing information and participating in surveys. The school’s parking lot and local shops are visited when collecting data. The community at large provides innumerable opportunities for budding mathematicians to practise their craft. Wherever possible, the mathematical ideas students are experiencing should be supported at home and given real-life significance, for example, cooking; buying clothes or toys and comparing prices; and calculating materials required for decorating.

**How does a PYP mathematics classroom work?**

Inquiry-based units of study are entry points into mathematics learning through which students will experience what it is like to think and act as mathematicians. Students and teachers together identify things they already know that might be relevant to an inquiry, what they want to know, what they need to know to answer their questions, and how best they might find that out. They are encouraged to use multiple strategies, developing an understanding of which strategies are most effective and efficient. The students must be given an opportunity to communicate their mathematical thinking and strategies to others and to have time to reflect upon them.

The teacher spends time in different ways: walking around while students are working alone, in pairs, in small groups or even as a whole class; asking key questions; challenging the students’ thinking and prompting them to take ideas a step further; and jotting down notes to inform the next stages of learning. The teacher might also gather together a group of students, with a particular interest or problem, to provide more specific help through guidance and practising together.

One of the most important aspects of the teacher’s role is to encourage appropriate mathematical discussion among the student mathematicians, demonstrating the nature of mathematical discourse and the development of conjectures. Students follow simple, polite rules when talking to each other, build on previously mentioned ideas, support others in the various stages of learning and share their discoveries in a congenial atmosphere. Students see writing down their ideas as a natural step in the process of communicating important ideas. They record their ideas in a variety of ways, including drawing pictures, recording numbers and writing in mathematics journals or log books.

An exemplary PYP mathematics classroom consists of a very active and busy community of learners, with the teacher constantly finding ways to combine the students’ needs and interests with the goals of the curriculum, using engaging and relevant tasks.

**Why is a PYP mathematics classroom the way it is?**

Mathematics is a language that is used to describe the natural world. Students need to build an understanding of previously made discoveries, as well as to discover and describe their own mathematical ideas.
Traditionally, mathematical knowledge has been disassembled in schools; broken into unrelated skills-based activities. Our vastly expanded knowledge base about learning mathematics, however, tells us that people assemble, or construct, mathematical knowledge. This requires us to look at mathematics not as a fixed body of knowledge to be transmitted, but as a language and a way of thinking. It is our task as teachers to facilitate this process.

Mathematics, like language, can be seen as a service discipline to other parts of the curriculum, providing tools, symbolic language and ways of thinking to the scientist, social scientist and musician; but mathematics is a fascinating discipline in its own right. It is the joy and satisfaction of solving problems and finding patterns that has captured and stimulated the most creative minds throughout the ages.

**How will we know what students have learned?**

Appropriate and regular assessment is essential in monitoring what students have learned. There should be ongoing formative assessments as well as summative assessments. Assessment activities should be carefully planned, and opportunities for students to self-assess, using different methods, should be included.

Examples of assessments are included in the scope and sequence document, along with activities and key questions for each expectation. English as additional language (EAL) learners may require special attention in order to fully participate in the activities. When the mathematical understandings of EAL learners are being assessed, care must be taken not to underestimate the students’ ability due to their lack of language proficiency.

The age bands of the scope and sequence document should be viewed only as a guide. A student’s ability to successfully understand and apply mathematical knowledge depends less on their age and more on the experience and support they receive. Some common difficulties have been described in the teachers’ notes where appropriate.

Record keeping should be simple and readily accessible for the teacher and the student. Examples of significant progress or development in mathematical understanding should be included in the student portfolio.

* See glossary for explanation of italicized terms.

**Recommended resources**


Glossary of PYP mathematical terms

acute angle
An angle of less than 90°.

addend
Any number added in an addition.

addition
Putting two or more sets together to create a new set. The inverse operation of subtraction. *Plus, and, increase by, more than, count on, find the total, what is the sum of?* can all be used to mean addition.

algebra
A mathematical language where relationships can be expressed in numbers, symbols or letters.

algebraic formula
A sequence of symbols used to express a rule or result.

algorithm
A method for calculating solutions.

angle
An amount of rotation, measured in degrees.

approximation
An estimation of a quantity that is close to the actual value but not totally accurate.

area
The amount of surface an object covers, usually measured in square units.

array
An arrangement showing quantities in rows and columns. Multiplication can be shown in this way to demonstrate square and rectangular numbers.

associative property
When the order in which mathematical operations are carried out does not affect the result. This applies to addition and multiplication, for example:

\[(2 + 3) + 4 = 2 + (3 + 4) = 9, \quad (5 \times 6) \times 7 = 5 \times (6 \times 7) = 210\]

axis (plural axes)
A line on a graph, usually either horizontal or vertical.

axis of symmetry
An imaginary line around which shapes are symmetrical. It can be a fold line in a 2-D shape or an axis around which shapes turn in rotational symmetry.

bar graph
A graph in which data is represented by horizontal or vertical bars.

base 10 system
A number system based on grouping in tens.
bearing

The position of something or direction of movement, relative to a fixed point. It is usually measured in degrees between the direction of travel and magnetic north, expressed as three figures.

It can also be expressed from north or south in the east or west direction. The angle in this case will always be less than 90°.

block graph

A graph in which data is shown by blocks, representing one unit, arranged in columns.

capacity

The space within a container, often referred to as volume.

cardinal number

A number denoting quantity (2, 3, 4) as opposed to order (1st, 2nd). See ordinal number.

Carroll diagram

A way of sorting any yes/no information using labelled boxes.

<table>
<thead>
<tr>
<th>less than 6</th>
<th>not less than 6</th>
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<td>7, 9</td>
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<tr>
<td></td>
<td>2, 4</td>
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<td>6, 8, 10</td>
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<table>
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<th>odd</th>
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</table>

<table>
<thead>
<tr>
<th>carrying</th>
<th>The process of marking an extra number to a column when adding or multiplying multiple digit numbers.</th>
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<tbody>
<tr>
<td>568</td>
<td>+ 354</td>
</tr>
<tr>
<td></td>
<td>922</td>
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<tr>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

Cartesian coordinates

Cartesian coordinates describe the position of a point on a graph or chart, so the lines of the axes must be numbered and not the spaces, also called a grid reference. See coordinates.
chord  A straight line, joining two points on the circumference of a circle, that does not pass through the centre of the circle.

circle  A round, plane shape whose boundary (circumference) is equidistant from its centre.

circle graph  See pie chart.

circumference  The distance around the outside of a circle, which can be calculated by multiplying the diameter by \( \pi \times d \).

commutative property  When the order in which numbers are arranged does not affect the result. This applies to addition and multiplication, for example:

\[
4 + 3 + 7 = 7 + 4 + 3 = 14, \quad 5 \times 3 \times 4 = 4 \times 3 \times 5 = 60
\]

complementary angles  Two angles whose sum is 90°.

congruency  The property of having the same size and shape.

congruent angles  Two angles of the same size.

conservation  The property of remaining the same despite changes such as physical arrangement.

coordinates  A pair of numbers (or a letter and number) placed in order to describe position. The first number describes the position on a horizontal axis, the second number describes the position on a vertical axis.

corner  Where two or more edges meet.

Cuisenaire rods  Commercially manufactured coloured rods of different sizes used to develop a sense of number facts and an understanding of fractions.

database  An organized and structured set of information, often held on a computer.

decimal notation  A way of showing values of less than 1 using place value and a dot to separate the whole units and fractional parts of a unit.

decomposition  See regrouping.

degree  1. A unit of measure for angles.
   2. A unit of measure for temperature (degrees centigrade, °C, used in the metric system).

denominator  The number below the line in a fraction, showing the number of equal parts in the whole.

\[
\frac{3}{4} \leftarrow \text{denominator}
\]

diagonal  A straight line passing through a shape and joining any two non-adjacent corners.

diameter  A line passing through the centre of a circle, with endpoints on the circumference.

difference  The result of a subtraction calculation.

digit (figure)  A symbol used to write numbers.
Glossary of PYP mathematical terms

**dividend**  
The amount to be divided in a division calculation.

**division**  
Breaking down a set into smaller sets of the same size as each other. The inverse operation of multiplication. The dividend is the amount to be divided, the divisor is the number of sets (or number in each set), and the quotient is the answer.

\[
\begin{align*}
45 & \div 9 = 5 \\
\uparrow & \quad \uparrow \\
\text{dividend} & \quad \text{divisor} \\
\text{quotient} &
\end{align*}
\]

**divisor**  
Number of equal sets or groups into which the dividend is divided.

**EAL**  
English as an additional language. Students learning in a language other than their mother tongue.

**edge**  
Where the two faces of a shape meet. It can be curved or straight.

**equation**  
A mathematical sentence balanced by an equals (=) sign. Equations can be numerical or algebraic.

**equilateral triangle**  
A triangle that has three equal sides and equal internal angles.

**equivalence**  
Expressing the same value in different ways, for example, \(0.75 = \frac{3}{4} = \frac{6}{8} = 75\%\).

**estimate**  
Arriving at a rough answer by making suitable approximations.

**experimental probability**  
When considering the likelihood of an event, it is what actually happens (how many times a six is thrown in 100 throws of a dice). See theoretical probability.

**exponential notation**  
A quick way of writing repeated multiplication, for example, \(6^3\), where 3 is the exponent, indicates that 6 is to be multiplied by itself 3 times \((6 \times 6 \times 6)\). \(6^3\) (6 to the power of 3) is in exponential notation. When the exponent is 2, the number is squared (squared number). When the exponent is 3, the number is cubed (cubed number).

**face**  
A surface of a 3-D shape.

**factor**  
A number that divides into another number exactly, for example, 8 is a factor of 8, 16, 24, 32 etc. The number 1 is a factor of all whole numbers.

**Fibonacci sequence**  
An infinite sequence in which each number is the sum of the two preceding numbers:  
1, 1, 2, 3, 5, 8, 13, 21, 34…

**formula**  
A set pattern for calculating a variable. It is usually written in algebraic form. For example, a formula to find the volume of a box is length \(\times\) width \(\times\) height so \(V = LWH\).

**fraction**  
A part of a whole. It can also be expressed as a percentage or a decimal.

**function**  
A relationship that should be applied to all numbers in a particular pattern, for example, add 2, divide by 3, double then subtract 1.
geoboard  A commercially manufactured plastic pinboard, used with elastic bands to make and explore shapes.

graph  A pictorial representation of information.

improper fraction  A fraction in which the numerator is greater than the denominator, for example, $\frac{7}{4}$, $\frac{9}{2}$.

integer  Any whole number, including those below zero.

inverse operations  Opposite operations, for example, addition and subtraction, multiplication and division.

isosceles triangle  A triangle that has two sides of equal length.

isometric dot paper  Paper with equally spaced dots.

kinaesthesia  An awareness of the position and movement of the parts of the body by means of sensory organs.

kite  A quadrilateral with two pairs of equal adjacent sides, perpendicular diagonals, and one line of symmetry. If one of the internal angles is reflex, the kite becomes an arrowhead.

line  The shortest distance between two points.

line graph  A graph that is used to represent the relationship between two continuous variables (for example, time and temperature). The line may be straight or curved.

line of symmetry  A line that divides a shape in half so that each side is a mirror image of the other.

manipulative  A concrete material.

mass  The amount of matter in an object. The term weight is often incorrectly used for mass.

mean  The arithmetic mean is a way of measuring the average in a set of data. It is calculated by finding the sum of the data and dividing by the number of sets of data, for example, in the data set 7, 9, 13, 18, 23 the mean is 14 ($7 + 9 + 13 + 18 + 23 = 70$; $70 \div 5 = 14$). Commonly referred to as the average.

median  The middle number in a set of data, for example, in the data set 7, 9, 13, 18, 23 the median is 13. In the data set 7, 9, 13, 15, 18, 23 the median is 14 ($13 + 15 = 28$; $28 \div 2 = 14$).

mixed number  A whole number and a fraction, for example, $3\frac{1}{2}$.

mode  The number that occurs most frequently in a set of data, for example, in the data set 3, 5, 6, 5, 7, 5, 8, 3, 5 the mode is 5.

model  To carry out the activity in a practical way using manipulatives. To demonstrate or show a method.

multiple  A natural number that can be divided by another, a certain number of times, without a remainder, for example, 12 is a multiple of 2, 3, 4, 6 and 12.
**multiplication**
A quick method of adding together groups of the same cardinality (repeated addition). The inverse function of division. The numbers to be multiplied are factors and the result is the product.

\[ 5 \times 9 = 45 \]

\[ \uparrow \quad \uparrow \quad \uparrow \]
\[ \text{factor} \quad \text{factor} \quad \text{product} \]

**natural number**
Any of the counting numbers (1, 2, 3, 4…).

**negative number**
Any number below zero.

**net**
A 2-D shape that can be folded to create a 3-D shape.

**non-standard units of measurement (arbitrary units)**
Any item used in the development of the concept of measurement, for example, feet, hand spans, paper clips or cubes.

**number facts**
Basic addition, subtraction, multiplication and division facts that students learn and use automatically.

**numerator**
The number above the line in a fraction, showing the number of parts being referred to.

\[ \text{numerator} \rightarrow \frac{3}{4} \]

**numerical probability scale**
The chance of an event happening is expressed between 0 and 1, or 0% and 100%, where 0 is impossible. Also, the term 50:50 expresses an even chance.

**obtuse angle**
An angle of more than 90° and less than 180°.

**one-to-one (1–1 correspondence**
Matching a number with an object. This can be by using numerals or simply by matching items to items (4 paper hats for 4 teddy bears).

**ordinal number**
A number that shows the relative position or place in a sequence: 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd}.

**parallel lines**
Lines that always stay the same distance apart and never meet.

**parallelogram**
A quadrilateral with opposite sides that are parallel and equal in length.

**pattern**
An arrangement of objects, shapes or numbers that repeats or changes in a regular way.

**percentage**
A way of expressing a fraction of an amount using a denominator of 100.

**perimeter**
The total distance around the outside of a 2-D shape.

**perpendicular lines**
Lines that cross at right angles.

**pictograph**
Data that is represented by pictures or symbols.

**pie chart**
A chart or graph in which a circle represents the whole and is then divided to represent the data.
place value
A method of recording numbers, using the digits in different positions to represent different values.

plane
A flat surface that can be vertical, horizontal or sloping.

polygon
A 2-D shape with many sides.

polyhedron
(plural polyhedra)
A 3-D shape with four or more faces.

prime number
Any number that has only two factors, 1 and itself. For example, 2, 3, 5, 7, 11… The number 1 is not a prime number as it only has one factor.

product
The answer to a multiplication calculation.

quadrants
The quarters of a grid. The first quadrant refers to the positive quarter of the grid.

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First quadrant

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quadrilateral
A polygon with four sides. They can be classified by their angles or their sides.

quotient
The answer to a division calculation.

radius
A line that has its endpoints in the centre of a circle and on the circumference. It is half the length of the diameter.

range
The spread of numbers in a set of data (the difference between the highest and lowest values). The range of numbers in the set of data 7, 9, 13, 18, 23 is 16 (23 – 7).

ratio
The relationship between two numbers or quantities.
Glossary of PYP mathematical terms

ray
A part of a line with only one endpoint.

real-life graph
A 3-D graph where each unit is represented by a single object.

rectangle
A parallelogram with four right angles (angles of 90°).

reflective symmetry
The exact matching of parts on either side of a straight line. It is sometimes called line symmetry, mirror symmetry or bilateral symmetry.

reflex angle
An angle of more than 180°.

regrouping
The exchange and rearrangement of tens for units, or hundreds for tens, in order to calculate multiple digit subtraction equations, for example:

\[
\begin{array}{c}
167 \\
- 59 \\
\hline
113
\end{array}
\]

The 7 tens have been “regrouped” into 6 tens and 10 units to allow the subtraction to take place. This method should only be used when a full understanding of place value has been achieved.

rhombus
A quadrilateral with two pairs of parallel sides, all sides of equal length, and opposite angles that are equal.

right-angled triangle
A triangle that has one angle of 90°.

roots
Equal factors of a product. Square root is two equal factors, cube root is three equal factors, for example, the square root of 81 is 9, \(\sqrt{81} = 9\) (9 \(\times\) 9 = 81). The cube root of 27 is \(3\sqrt[3]{27} = 3\) (3 \(\times\) 3 \(\times\) 3 = 27).

rotational symmetry
When a shape fits onto itself after a turn of less than 360°, it is said to have rotational symmetry. The number of turns required to return to its original position describe the order of symmetry. For example, a square has a rotational symmetry of order 4.

scale
The relative size or extent of something, a ratio of size.

scalene triangle
A triangle that has three sides of different lengths.

sector
A slice of a circle.

segment
A part of a circle bound by a chord and the circumference.

set, subset, superset
A way of grouping and classifying numbers or objects, for example, a set may be cats, a subset would be all black cats and a superset would be all pets.

side
One of the lines forming the boundary of a plane shape.

spreadsheet
A type of database where information is set out in a table.

square
A rectangle with sides of equal length.

standard units of measurement
Internationally recognized units of measurement, for example, centimetres, inches, kilograms, seconds, minutes.
straight angle  
An angle of 180°.

subtraction  
The inverse operation of addition. It calculates the difference between two amounts.  
*Minus, take away, decrease by, less than, fewer than, the difference between, count back* can also be used to mean subtract.

sum  
The result of adding numbers together, also referred to as the total.

supplementary angles  
Two angles whose sum is 180°.

surface area  
The sum of the area of the faces of a 3-D shape.

survey  
A method of obtaining information, usually by asking people questions.

symmetry  
Having exactly matching parts facing each other around a line or axis.

t-chart  
A chart that displays information before and after a function has been applied.

<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
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<tbody>
<tr>
<td>2</td>
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<td>4</td>
<td>8</td>
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<tr>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

(two function is +4)

tessellation  
A tiling pattern where shapes fit together without overlapping or gaps.

theoretical probability  
When considering the likelihood of an event, it is what is *expected* to happen (how many times 6 *might* be thrown in 100 throws of a dice). See experimental probability.

three-dimensional (3-D) shape  
A solid shape that can be measured in three dimensions – length, width and height. Examples include sphere, cylinder, cone, prism, pyramid and poyhedron.

total  
The result of adding numbers together, also referred to as the sum.

trapezium (UK)/trapezoid (US)  
A quadrilateral with one pair of parallel sides.

tree diagram  
A branching diagram to show all possible outcomes.

triangle  
A plane shape with three straight sides.

turn symmetry  
See rotational symmetry.

two-dimensional (2-D) shape  
A flat shape that can be measured in two dimensions (length and width). Examples include circle, square, triangle and polygon.
Glossary of PYP mathematical terms

**Venn diagram**
A way of displaying data sorted by characteristics. Where the items may belong in two of the groupings, the groupings overlap.

**vertex (plural vertices)**
A point where lines or edges meet.

**volume**
The amount of space a solid shape occupies. It is usually measured in cubic units.

**weight**
The measure of the heaviness of an object. The term is often incorrectly used for mass. (It is calculated by multiplying mass by gravity, which explains why astronauts are “weightless” even though their mass remains the same.)

**whole number**
Any of the natural numbers, together with zero (0, 1, 2, 3, 4…).
Mathematics scope and sequence overview

In addition to the following strands, students will have the opportunity to identify and reflect upon “big ideas” by making connections between the questions asked and the concepts that drive the inquiry. They will become aware of the relevance these concepts have to all of their learning.

<table>
<thead>
<tr>
<th>Strand</th>
<th>By the end of this age range, children aged 3–5 will:</th>
<th>By the end of this age range, students aged 5–7 will:</th>
<th>By the end of this age range, students aged 7–9 will:</th>
<th>By the end of this age range, students aged 9–12 will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data handling</td>
<td>• sort and label real objects into sets by attributes</td>
<td>• sort and label objects into sets by one or more attribute</td>
<td>• discuss, compare and create sets from data that has subsets using tree, Carroll, Venn and other diagrams</td>
<td>• design a survey, process and interpret the data create a graph of real objects and compare quantities using number words collect, display and interpret data for the purpose of finding information understand the purpose of graphing data create a pictograph and simple bar graph from a graph of real objects, and interpret data by comparing quantities: more, fewer, less than, greater than discuss, identify, predict and place outcomes in order of likelihood: impossible, unlikely, likely and certain.</td>
</tr>
<tr>
<td></td>
<td>• identify, compare and describe attributes of real objects and situations: longer, shorter, heavier, empty, full, hotter, colder</td>
<td>• identify, compare and sequence events in their daily routine: before, after, bedtime, storytime, today, tomorrow.</td>
<td>• estimate, measure, label and compare using non-standard units of measurement: length, mass, time and temperature understand why we use standard units of measurement to measure use a calendar to determine the date, and to identify and sequence days of the week and months of the year estimate, identify and compare lengths of time: second, minute, hour, day, week, month read and write the time to the hour, half hour and quarter hour</td>
<td>• select and use appropriate standard units of measurement when estimating, describing, comparing and measuring using measuring tools, with simple scales, accurately understand that the accuracy of a measurement depends on the situation and the precision of the tools develop procedures for finding area, perimeter and volume determine the relationships between area, perimeter and volume estimate, measure, label and compare, using formal methods and standard units of measurement, the dimensions of area, perimeter and volume use decimal notation in measurement: 3.2cm, 1.47kg understand that an angle is a measure of rotation measure and construct angles in degrees using a protractor use and construct timetables (12-hour and 24-hour) and time lines determine times worldwide.</td>
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<td>• identify, compare and describe attributes of real objects and situations: longer, shorter, heavier, empty, full, hotter, colder</td>
<td>• identify, compare and sequence events in their daily routine: before, after, bedtime, storytime, today, tomorrow.</td>
<td>• estimate, measure, label and compare using formal methods and standard units of measurement: length, mass, time and temperature select appropriate tools and units of measurement describe measures that fall between numbers on a measure scale: 3/4kg, between 4cm and 5cm estimate, measure, label and compare perimeter and area model addition and subtraction using money read and write the time to the minute and second, using intervals of 10 minutes, 5 minutes and 1 minute, on 12-hour and 24-hour clocks.</td>
<td>• select and use appropriate standard units of measurement when estimating, describing, comparing and measuring using measuring tools, with simple scales, accurately understand that the accuracy of a measurement depends on the situation and the precision of the tools develop procedures for finding area, perimeter and volume determine the relationships between area, perimeter and volume estimate, measure, label and compare, using formal methods and standard units of measurement, the dimensions of area, perimeter and volume use decimal notation in measurement: 3.2cm, 1.47kg understand that an angle is a measure of rotation measure and construct angles in degrees using a protractor use and construct timetables (12-hour and 24-hour) and time lines determine times worldwide.</td>
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<td>• sort, describe and compare 3-D shapes according to attributes such as size or form</td>
<td>• use what they know about 3-D shapes to see and describe 2-D shapes sort and label 2-D and 3-D shapes using appropriate mathematical vocabulary: sides, corners, circle, sphere, square, cube create 2-D shapes find and explain symmetry in their immediate environment create and explain simple symmetrical designs give and follow simple directions, describing paths, regions and boundaries of their immediate environment and their position (next to, behind, in front of, up, down).</td>
<td>• sort, describe and model regular and irregular polygons: triangles, hexagons, trapeziums identify, describe and model congruency in 2-D shapes combine and transform 2-D shapes to make another shape create symmetrical patterns, including tessellation identify lines and axes of reflective and rotational symmetry understand an angle as a measure of rotation by comparing and describing rotations: whole turn; half turn; quarter turn; north, south, east and west on a compass locate features on a grid using coordinates.</td>
<td>• use the geometric vocabulary of 2-D and 3-D shapes: parallel, edge, vertex classify, sort and label all types of triangles and quadrilaterals: scalene, isosceles, equilateral, right-angled, rhombus, trapezium, parallelogram, kite, square, rectangle understand and use the vocabulary of types of angle: obtuse, acute, straight, reflex understand and use geometric vocabulary for circles: diameter, radius, circumference use a pair of compasses understand and use the vocabulary of lines, rays and segments: parallel, perpendicular describe, classify and model 3-D shapes turn a 2-D net into a 3-D shape and vice versa find and use scale (ratios) to enlarge and reduce shapes use the language and notation of bearing to describe position read and plot coordinates in four quadrants.</td>
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<tr>
<td><strong>Pattern and function</strong></td>
<td>• find and describe simple patterns</td>
<td>• create, describe and extend patterns</td>
<td>• analyse patterns in number systems to 100</td>
<td>• understand and use the relationship between multiplication and addition</td>
</tr>
<tr>
<td></td>
<td>• create simple patterns using real objects.</td>
<td>• recognize, describe and extend patterns in numbers: odd and even, skip counting, 2s, 5s and 10s</td>
<td>• recognize, describe and extend more complex patterns in numbers</td>
<td>• understand and use the relationship between multiplication and division (inverse function)</td>
</tr>
<tr>
<td></td>
<td>• identify patterns and rules for addition: 4 + 3 = 7, 3 + 4 = 7 (commutative property)</td>
<td>• identify patterns and rules for addition: 7 - 3 = 4, 7 - 4 = 3</td>
<td>• identify patterns and rules for multiplication and division: 4 × 3 = 12, 3 × 4 = 12, 12 ÷ 4 = 3, 12 ÷ 3 = 4</td>
<td>• understand and use the relationship between division and subtraction</td>
</tr>
<tr>
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<td>• model, with manipulatives, the relationship between addition and subtraction: 3 + 4 = 7, 7 - 3 = 4.</td>
<td>• model, with manipulatives, the relationship between multiplication and division</td>
<td>• model, with manipulatives, the relationship between multiplication and addition (repeated addition)</td>
<td>• model and explain number patterns</td>
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<td>• model, with manipulatives, the relationship between addition and subtraction: 3 + 4 = 7, 7 - 3 = 4.</td>
<td>• model multiplication as an array</td>
<td>• model multiplication as an array</td>
<td>• use real-life problems to create a number pattern, following a rule</td>
</tr>
<tr>
<td></td>
<td>• model multiplication as an array</td>
<td>• understand and use number patterns (missing numbers).</td>
<td>• understand and use number patterns (missing numbers).</td>
<td>• develop, explain and model simple algebraic formulas in more complex equations: x + 1 = y, where y is any even whole number</td>
</tr>
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<td>• model, with manipulatives, the relationship between addition and subtraction: 3 + 4 = 7, 7 - 3 = 4.</td>
<td>• model, with manipulatives, the relationship between addition and subtraction: 3 + 4 = 7, 7 - 3 = 4.</td>
<td>• use exponents as repeated multiplication</td>
<td>• model exponents as repeated multiplication</td>
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<td>• model, with manipulatives, the relationship between multiplication and addition (repeated addition)</td>
<td>• model multiplication as an array</td>
<td>• model exponents as repeated multiplication</td>
<td>• understand and use exponents and roots as inverse functions: 9ⁿ, ½ⁿ</td>
</tr>
<tr>
<td><strong>Number</strong></td>
<td>• read, write and model numbers to 20</td>
<td>• read, write, and model numbers, using the base 10 system, to 100</td>
<td>• read, write and model numbers, using the base 10 system, to 100</td>
<td>• read, write and model numbers, using the base 10 system, to millions and beyond; and to thousands and beyond</td>
</tr>
<tr>
<td></td>
<td>• count, compare and order numbers to 20</td>
<td>• count (in 1s, 2s, 5s and 10s), compare and order numbers to 100</td>
<td>• estimate quantities to 1000</td>
<td>• automatically recall and use basic number facts</td>
</tr>
<tr>
<td></td>
<td>• estimate quantities to 10</td>
<td>• estimate quantities to 100</td>
<td>• automatically recall addition and subtraction facts to 10</td>
<td>• create and solve multiple digit multiplication and division problems</td>
</tr>
<tr>
<td></td>
<td>• use ordinal numbers to describe the position of things in a sequence</td>
<td>• use mathematical vocabulary and symbols of addition and subtraction: add, subtract, difference, sum, +, –</td>
<td>• use mathematical vocabulary and symbols of addition and subtraction: add, subtract, division, quotient, x, ÷</td>
<td>• read, write and model addition and subtraction of fractions with related denominators</td>
</tr>
<tr>
<td></td>
<td>• model number relationships to 10: “Show me one more than three, take two away from these cubes”</td>
<td>• read, write and model addition and subtraction to 20 (with and without regrouping)</td>
<td>• read, write and model addition and subtraction to 20 (with and without regrouping)</td>
<td>• read, write and model improper fractions and mixed numbers</td>
</tr>
<tr>
<td></td>
<td>• use the language of mathematics: more, less, number names, total</td>
<td>• automatically recall addition and subtraction facts to 10</td>
<td>• automatically recall basic addition and subtraction facts</td>
<td>• compare and order fractions</td>
</tr>
<tr>
<td></td>
<td>• use 1–1 correspondence</td>
<td>• describe the meaning and use of addition and subtraction</td>
<td>• model addition and subtraction equations to 1000 (with and without regrouping)</td>
<td>• model equivalency of fractions: ½ = 1/2</td>
</tr>
<tr>
<td></td>
<td>• explore the conservation of number through the use of manipulatives</td>
<td>• explore and model multiplication and division using their own language/methods</td>
<td>• use mathematical vocabulary and symbols of multiplication and division: times, divide, product, quotient, ÷, x</td>
<td>• simplify fractions</td>
</tr>
<tr>
<td></td>
<td>• select and explain an appropriate method for solving a problem.</td>
<td>• use fraction names (half, quarter) to describe part and whole relationships</td>
<td>• use and describe multiple strategies to solve addition, subtraction, multiplication and division problems</td>
<td>• use the mathematical vocabulary of fractions: improper, mixed numbers</td>
</tr>
<tr>
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<td>• estimate the reasonableness of answers</td>
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<td>• read, write and model multiplication and division problems</td>
<td>• read, write and model the addition and subtraction of decimals to the thousands</td>
</tr>
<tr>
<td></td>
<td>• select and explain an appropriate method for solving a problem.</td>
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<td>• compare fractions using manipulatives and using fractional notation</td>
<td>• read, write and model multiplication and division of decimals (with reference to money)</td>
</tr>
<tr>
<td></td>
<td>• use mathematical vocabulary and symbols of fractions: numerator, denominator, equivalence</td>
<td>• model addition and subtraction of fractions with the same denominator</td>
<td>• use mathematical vocabulary and symbols of fractions: numerator, denominator, equivalence</td>
<td>• round decimals to a given place or whole number</td>
</tr>
<tr>
<td></td>
<td>• understand and model the concept of equivalence to 1: two halves = 1, three thirds = 1</td>
<td>• use mathematical vocabulary and symbols of fractions: numerator, denominator, equivalence</td>
<td>• understand and model the concept of equivalence to 1: two halves = 1, three thirds = 1</td>
<td>• read, write and model percentages</td>
</tr>
<tr>
<td></td>
<td>• reasonably estimate answers: rounding and approximation</td>
<td>• understand and model the concept of equivalence to 1: two halves = 1, three thirds = 1</td>
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<td>• interchange fractions, percentages and decimals</td>
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<td></td>
<td>• select and explain an appropriate method for solving a problem.</td>
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<td>• find and use ratios</td>
</tr>
<tr>
<td></td>
<td>• use mathematical vocabulary and symbols of fractions: numerator, denominator, equivalence</td>
<td>• read, write and model integers</td>
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</tr>
<tr>
<td></td>
<td>• understand and model addition and subtraction of integers (negative numbers)</td>
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<td>• read, write and model exponential notation</td>
<td>• use real-life problems to create a number pattern, following a rule</td>
<td>• read, write and model exponential notation</td>
<td>• select and defend the most appropriate and efficient method of solving a problem: mental estimation, mental arithmetic, pencil and paper algorithm, calculator</td>
</tr>
</tbody>
</table>
Subject: mathematics  
Age range: 3–5 years  
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### Overall expectations
Children will have the opportunity to identify and reflect upon “big ideas” by making connections between the questions asked and the concepts that drive the inquiry. They will become aware of the relevance these concepts have to all of their learning.

### Data handling: statistics and probability
Children will sort real objects by attributes, create graphs using real objects and compare quantities. They will discuss and identify outcomes that will happen, won’t happen or might happen.

### Measurement
Children will identify and compare attributes of real objects, and events in their realm of experience.

### Shape and space
Children will sort, describe and compare 3-D shapes and explore the paths, regions and boundaries of their immediate environment and their position.

### Pattern and function
Children will find, describe and create simple patterns in their world.

### Number
Children will read, write, count, compare and order numbers to 20. They will model number relationships to 10, develop a sense of 1–1 correspondence and conservation of number. They will select and explain an appropriate method for solving a problem.

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### Content | What do we want children to learn? | How best will children learn? | How will we know what children have learned? | Notes for teachers
---|---|---|---|---

**Specific expectations**

The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

**Sample questions**

Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

**Sample activities**

All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen).

**Sample assessments**

Assessments should be directly related to the specific expectations. Children should be given the opportunity to demonstrate their understanding in a variety of ways.

### Data handling: statistics

Data can be recorded, organized, displayed and understood in a variety of ways to highlight similarities, differences and trends.

**Children will:**

- sort and label real objects into sets by attributes
- create a graph of real objects and compare quantities using number words.

**Questions that address the key concepts**

- What things belong together?
- Why do they belong together?
- Can we sort them in another way?

**Sample questions**

- What different ways are there to show what we have done?
- Which has more?
- Which has the most?
- Which has the least?
- Can we see any patterns in the graph?
- How can we describe the information in these graphs?
- How can we use number words to describe what we see on the graph?

**Sample activities**

- Children sort collections of keys, nuts, bottle tops, blocks and themselves to determine attributes.
- Children physically group themselves by eye colour or shoe colour.

**Sample assessments**

- Children are presented with a set of objects and can sort them in two different ways using attributes they have identified. They can justify their criteria.

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* See glossary for explanation of italicized terms.
### Data handling: probability

**Appropriate qualitative vocabulary** may be introduced such as “will happen”, “won’t happen” and “might happen”, as guided by the exploration and the experiences of the children.

**Children will:**
- discuss and identify outcomes that will happen, won’t happen and might happen.

**Probability questions**
*Children are asked a series of questions: Will an elephant walk in through the classroom door? Will it snow in mid-summer? Will it be sunny in mid-summer? Will it be dark at lunchtime? Will it be light at lunchtime? Will it be your birthday tomorrow? Will it be Thursday tomorrow?*

**In discussions about the weather, children are asked a series of questions related to probability:** Will it rain today? Will it snow today? Do you think the sun will shine all day? Will this storm stop?

### Measurement

**To measure is to attach a number to a quantity using a chosen unit.** However, since the attributes being measured are continuous, ways must be found to deal with quantities that fall between the numbers. It is important to know how accurate a measure needs to be or can ever be.

**Children are given real objects and situations in which they must compare length, mass, duration and temperature.**

**Children are asked to determine which is longer, taller, heavier, hotter, larger and can demonstrate direct comparisons:** “My pencil is longer than his”, “Her bag is heavier than mine.”

**Children are given opportunities to directly compare three or more objects of unequal length.**

**Children are asked a series of questions related to probability:** Will it rain today? Will it snow today? Do you think the sun will shine all day? Will this storm stop?

**Children can identify which of these things will happen, won’t happen or might happen and they can suggest a reasonable explanation why.**

**Children can identify outcomes and can justify their predicted outcomes in simple terms.**

**Children can identify outcomes and can justify their predicted outcomes in simple terms.**

**Children should also be given objects of equal length to lead to the idea of equivalence.** Are objects still the same length even when they are not lined up next to each other?

**Children can sequence events correctly and can explain that it will be time to go home after a story, time for bed after bathtime etc.**

**Children can sequence events correctly and can explain that it will be time to go home after a story, time for bed after bathtime etc.**

### Resources and comments

- **Teachers should find ways to ensure EAL learners understand tasks and expectations.** *Manipulatives facilitate understanding for all learners.*
- **Questions such as these will depend upon the location and the time of year.**

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### Shape and space

Children need to understand the interrelationships of shapes, and the effects of changes to shape, in order to understand, appreciate, interpret, and modify our two-dimensional and three-dimensional world.

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<td><strong>Children will:</strong></td>
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<td></td>
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</tr>
<tr>
<td>• sort, describe and compare 3-D shapes according to attributes such as size or form</td>
<td></td>
<td><strong>Exploring 3-D shapes</strong></td>
<td>Children are given a variety of 3-D objects that may include geometric shapes. They are encouraged to look at the objects in a variety of ways, and from different directions, to identify similarities and differences. Children are given a variety of shapes and objects to explore by touching, observing and talking about them. Children explore whether objects/shapes roll or stack.</td>
<td></td>
</tr>
<tr>
<td><strong>The regions, paths and boundaries of natural space can be described by shape.</strong></td>
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<td><strong>Pattern and function</strong></td>
<td>To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called functions. This builds a foundation for the later study of algebra.</td>
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#### Shape and space

- Children will:
  - explore and describe the paths, regions and boundaries of their immediate environment (inside, outside, above, below) and their position (next to, behind, in front of, up, down).
  - find and describe simple patterns

- **Regions and boundaries**
  - Children use ropes, walls etc to explore regions and boundaries and their position in relation to these objects.
  - In PE, children stand behind, in front of or next to a partner.
  - Children use rubber bands and geoboards to create shapes with defined boundaries.

- **Finding patterns**
  - Children can use their own explanations to describe the patterns they see and can identify and describe the repetitive features.
  - Children can predict the next step in a pattern.

- **Children can use their own explanations to describe the patterns they see and can identify and describe the repetitive features.**

  - Children identify the common features of each pattern and are able to compare the patterns to identify similarities and differences.

- **What is a pattern? form**
  - Children recognize patterns in animals: stripes, spots, feathers, wings of butterflies.
  - Children recognize patterns in carpets, architecture and common household objects.
  - Children sort, match and compare patterns on scraps of wallpaper or fabric.

- **Can we describe the shapes and talk about how they worked?**

  - Children can sort and describe the shapes and begin to classify them according to the shapes’ attributes.

- **Children can name some of the shapes and begin to classify them according to the shapes’ attributes.**

- **What is outside?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Children can use their own language to describe their position in relation to others.**

- **What is inside?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What is different about inside and outside?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What is the same and what is different about inside and outside?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Does a shape change if its shape changes?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Does it always look the same down/over?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What is inside?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What is the same about shapes?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What is different about them? function**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What can you tell me about your shape? reflection**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Why is a block the best shape for building a tower? causation**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Can we make this shape from other shapes? causation**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Does a shape change if its size changes?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Does a shape change if its shape changes?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Does it always look the same when you turn it upside down/over?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **Pattern and function**
  - To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called functions. This builds a foundation for the later study of algebra.

- **What is the same about these shapes?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What is different about them? function**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What is the same about shapes that are flat/curved?**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **How can we use these shapes? responsibility**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.

- **What can you tell me about your shape? reflection**
  - Children can use everyday language to describe what is inside, outside, next to, in front of and behind in a given situation.
### Content

#### Pattern and function (cont.)

Children will:
- create simple patterns using real objects.

Questions that address the key concepts (Fig. 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

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<td>The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.</td>
<td>What pattern can we make with these shells, buttons, blocks? function Can we make a different pattern with them? Can we make up a rhythm pattern using our hands and feet? function</td>
<td>Creating patterns Children use blocks, colour tiles, buttons or keys to copy and create simple patterns. Create a rhythm pattern using clapping, clicking fingers, stamping feet. Children begin to hear simple rhythms in music.</td>
<td>Assessments should be directly related to the specific expectations. Children should be given the opportunity to demonstrate their understanding in a variety of ways.</td>
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#### Number

Our number system is a language for describing quantities and relationships between quantities. The value attributed to a digit depends on its place within a base system. The operations of *addition, subtraction, multiplication* and *division* are related to one another and are used to process information in order to solve problems.

- read, write and model numbers to 20

#### Specific expectations

Children can recognize numbers in their immediate environment and can read the numbers on a clock face or on a computer keyboard.

**Sample questions**

- What numbers do we know?
- Where do we find numbers?

**Sample activities**

**Number walk**

- Take the class on a number walk around the classroom, school buildings or school grounds. Children start to point out numbers in the immediate environment and use them to create a display of “numbers we can see.”

**Reading and writing numbers**

- Children use numeral cards to label sets of objects.
- Children draw numbers in sand with their fingers, paint numbers and make numbers using modelling clay.

**Counting activities**

- Children count objects arranged in a line or randomly.
- Children count drum beats or claps in a rhythm.
- Children count the number of jumps it takes to cross the room.
- Children count and compare the letters of their names.

**Number rhymes and songs**

- Children join in with rhymes and songs such as *Five Little Speckled Frogs, Ten Green Bottles* and *One Man Went To Mow.*
- Rockets taking off require a countdown. Children start from numbers other than ten and use zero as part of the sequence.

**Sample assessments**

- Children can recognize numbers in their immediate environment and can read the numbers on a clock face or on a computer keyboard.
- Children can write the number that is represented when they have counted objects.

**Resources and comments**

- The teacher could make number labels for seats, coat pegs etc for children to find on their number walk.
- Assessment will be oral in nature.
- Children should have richly varied opportunities for counting real objects that are related to their studies.
- A wide variety of everyday materials should be used, as well as some mathematics manipulatives such as Unifix cubes.
- There are many finger and action rhymes suitable for developing an awareness of numbers and counting.
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<td>Children will:</td>
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<td>• estimate quantities to 10</td>
<td>How is estimating the same as counting?</td>
<td>Children can make reasonable estimates of small groups of objects without counting.</td>
<td>Compare the ease of estimating when children can/cannot see the objects.</td>
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<td>How is estimating different from counting?</td>
<td>They can check their estimates and refine it if necessary, after counting.</td>
<td>Estimation is a skill that will develop with experience and will help children gain a “feel” for numbers. Children must be given the opportunity to check their estimates so that they are able to further refine and improve their estimation skills.</td>
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<td>How many objects do you think I have in my (closed) hand?</td>
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<td>Try to find authentic contexts in your units of inquiry to practise estimating up to 20.</td>
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<td>How many objects do you think I have in my (open) hand now?</td>
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<td></td>
<td>Did you change your estimate? Why?</td>
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<td></td>
<td>Which is easier to estimate? Why?</td>
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<td>• use ordinal numbers to describe the position of things in a sequence</td>
<td>How can we describe the order in which these things, objects, children are arranged?</td>
<td>Children can create pictures that accurately represent the information in the story.</td>
<td>Arrange athletic activities (jumping, throwing, running events) to allow children to use the words “first”, “second”, “third” in context.</td>
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<td>Children can describe orally the order in which they arrived in the room, using the correct ordinal numbers.</td>
<td>For a child who knows cardinal numbers, learning ordinal numbers is primarily a language issue.</td>
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<td>Children will also have to demonstrate an understanding of positional language: before, after, between, next.</td>
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<td>• model number relationships to 10: “Show me one more than three, take two away from these cubes”</td>
<td>What connections are there between these numbers?</td>
<td>Children can show the different ways in which they can make addition sentences using manipulatives that equal 4, 8, 10.</td>
<td>Good number sense is based on richly interrelated number meaning.</td>
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<tr>
<td></td>
<td>connection</td>
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<td>Mental arithmetic, including the basic facts, is easier if children have had experience of deconstructing numbers: $8 - 5 = 3 = 6 + 2$.</td>
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<td>How does the number change when we add one more? change</td>
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<tr>
<td>• use the language of mathematics: more, less, number names, total</td>
<td>What number names do we know?</td>
<td>Children can use these new terms in their discussions.</td>
<td>From the earliest stages, correct and appropriate mathematical language should be used. Consistent use will enable children to remember, and use, relevant mathematical vocabulary with confidence.</td>
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<td>How will we count these? How can we be sure that we have the right number of plates/cups?</td>
<td>Children match objects using 1–1 correspondence.</td>
<td>Children can lay the table to correspond to one plate or cup or set of cutlery for each person.</td>
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<td>Children set the table for four friends in the home corner.</td>
<td>One to one matching Children match objects using 1–1 correspondence.</td>
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<td>Children can identify the fact that there are still three goats regardless of their position.</td>
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<td>Children will:</td>
<td>• use 1–1 correspondence</td>
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<td>• explore the conservation of number through the use of manipulatives [● ● ● ● ●  ● ● ]</td>
<td>How many objects are here? How many are there now? Is it the same number? How do you know?</td>
<td>Conservation of number Act out the story of The Three Billy Goats Gruff. Children can see that even when one of the goats “trip-traps” over the bridge, there are still three goats. Ask six children (or any other number) to stand in a group. They may be holding numeral cards. When they are bunched together, count and identify all six members of the group. Ask the children to spread out from each other and count again. They can regroup in any way to demonstrate the conservation of number. This activity can be repeated with a variety of different objects.</td>
<td>Children can conserve, in mathematical terms, means the amount stays the same regardless of the arrangement. Young children are often fooled by what their eye sees when objects are rearranged. Lots of counting and rearranging of manipulatives will help in overcoming this. Children cannot be taught how to conserve, but an understanding is reached through reason and experience. Conservation can also apply to measurement relationships.</td>
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<td>• select and explain an appropriate method for solving a problem.</td>
<td>How can we find an answer? Are there any other ways to find out? What (materials) can we use to help us? Do we always find the same answer even when we do things in a different way? change Which way works the best? Why? reflection</td>
<td>Doing it my way Give the children a problem, and ask them to select materials to help them solve it. Children explain why they chose those materials.</td>
<td>At this stage of development, the method selected may not be the most effective. Children should have regular opportunities to select and explain their ideas and methods. Children who have been encouraged to select their own apparatus and methods, and who become accustomed to discussing and questioning their work, will have confidence in looking for alternative approaches when an initial attempt is unsuccessful. Children notice and discuss difficulties and make suggestions to one another.</td>
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### Overall expectations

Students will have the opportunity to identify and reflect upon “big ideas” by making connections between the questions asked and the concepts that drive the inquiry. They will become aware of the relevance these concepts have to all of their learning.

### Data handling: statistics and probability

**Specific expectations**

The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

**Sample questions**

- Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding.
  - All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

**Sample activities**

- All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen).

**Sample assessments**

- Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.

**Data handling: statistics**

- Data can be recorded, organized, displayed and understood in a variety of ways to highlight similarities, differences and trends.

- Students will:
  - sort and label objects into sets by one or more attribute
  - discuss and compare data represented in teacher-generated diagrams: tree, Carroll, Venn

- Why would we want to use diagrams to represent our data?
- What information can we tell by looking at these diagrams?
- How can we tell that data belongs in more than one set?
- How do we know if the same information is represented in both of these diagrams?

- Students are given a Venn diagram and a Carroll diagram representing the same information. They can explain whether or not the diagrams represent the same data and how they know this.

### Shape and space

Students will describe the properties of 3-D shapes, including the 2-D shapes that can be seen, using mathematical vocabulary. They will find and explain symmetry in the immediate environment and create symmetrical patterns. They will give and follow simple directions using left, right, forward and backward.

### Pattern and function

Students will describe, continue, create and compare patterns. They will recognize and extend patterns in number. They will identify commutative property. They will model the relationships in, and between, addition and subtraction.

### Number

- Students will read, write, estimate, count, compare and order numbers to 1000. They will read, write, model and understand addition and subtraction, using mathematical vocabulary and symbols. They will automatically use addition and subtraction facts to 10. They will explore multiplication and division using their own methods, use fraction names to describe part and whole relationships, and explore counting patterns. They will select and explain appropriate methods for solving a problem and estimate reasonableness of answers.

- Students will require a lot of practice of sorting and labelling objects into sets by more than one attribute.

- Students will have a lot of hands-on experience of sorting and labelling objects into sets by one common attribute.

- Students look at the various attributes of each of the pattern blocks.

- As each pattern block has a specific size, shape and colour, students can sort them by more than one attribute.

- Students will have to identify and reflect upon “big ideas” by making connections between the questions asked and the concepts that drive the inquiry. They will become aware of the relevance these concepts have to all of their learning.

### Notes for teachers

- Teachers should find ways to ensure EAL learners understand tasks and expectations.

- Manipulatives facilitate understanding for all learners.

- At this stage of development, students have a lot of hands-on experience of sorting and labelling objects into sets by one common attribute.

- Students will require a lot of practice of sorting and labelling objects into sets by more than one attribute.

- “Everything in this set is round and blue.”

### Resources and comments

- Teachers should find ways to ensure EAL learners understand tasks and expectations.

- Students will have a lot of hands-on experience of sorting and labelling objects into sets by one common attribute.

- As each pattern block has a specific size, shape and colour, students can sort them by more than one attribute.

- Students will require a lot of practice of sorting and labelling objects into sets by more than one attribute.

- “Everything in this set is triangular and green.”

- It is easier for students to understand and interpret sets and subsets that intersect if somebody else has created them. Being able to create diagrams, subsets and intersecting sets is more complex and challenging.

- Computer programs that require users to manipulate data in order to create graphic organizers are excellent tools for developing an understanding of these mathematical concepts.
### Data handling: statistics (cont.)

**Students will:**

- collect, display and interpret data for the purpose of finding information

**What do we want students to learn?**

- understand the purpose of graphing data

**How best will students learn?**

**Sample questions**

The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

- **Collecting and displaying data**
  - Students work in groups to find the answer to a question: How many students have pets? What pets do they have? Students work in groups to collect and record the data in any form.
  - Students display their data in two different formats: list, tally marks. They exchange their data displays with another group and interpret the findings.

**Sample activities**

- **Why would we want to collect data?**
  - Why would we want to collect data?
  - What questions will give us the information we need?
  - How can we display data?
  - What information can we tell by looking at these diagrams?
  - How can we tell that data belongs in more than one set?
  - How do we know if the same information is represented in both of these diagrams?

**Sample assessments**

- **Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.**

**How will we know what students have learned?**

- **Students are given a set of data. They can interpret the information and explain how the information can be used.**
  - Students can represent the same data in a different format.

**Resources and comments**

- Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.

**What are the important features of a graph?**

- How did we reach our interpretations? reflection

**Creating simple graphs**

- Students sort and label their snacks into different categories and use the empty wrappers to create a real-life graph based on their findings. Students use the real-life graph to create a bar graph representing the same data. They provide a title for their graph and label each axis. Students interpret the data and compare quantities.

**Sample questions**

- How do we know we can keep a record of our real-life graph?
  - How are graphs used?
  - How are graphs the same as, and different from, lists and other types of diagram?
  - What are the important features of a graph?
  - How did we reach our interpretations?

**Sample activities**

- **Creating simple graphs**
  - Students sort and label their snacks into different categories and use the empty wrappers to create a real-life graph based on their findings. Students use the real-life graph to create a bar graph representing the same data. They provide a title for their graph and label each axis. Students interpret the data and compare quantities.

**Sample assessments**

- **Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.**

**Notes for teachers**

- Students need to think about the data they have collected and how they can organize it in the most effective and efficient way. They should have a lot of experience organizing data in a variety of ways, and of talking about the advantages and disadvantages of each.

- Interpretations of data should include the information that cannot be concluded as well as that which can. It is important to remember that the chosen format should illustrate the information without bias.

**What is the purpose of finding information?**

- The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

**How are other types of diagrams used?**

- These can be simply viewed as the frame of a pictograph.

**Sample questions**

- How are graphs, simple pictographs and bar graphs? They are asked to look for similarities and differences between the features and set-up of each graph.

**Sample assessments**

- Students can represent the same data in a different format.
### Data handling: probability

There are ways to find out if some outcomes are more likely than others. Probability can be expressed qualitatively by using terms such as certain, likely, unlikely or impossible.

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### Measurement

To measure is to attach a number to a quantity using a chosen unit. However, since  the attributes being measured are continuous, ways to deal with quantities that fall between the numbers must be found. Also, by attaching a number to a quantity, students can begin to understand how mathematics applies to the real world.

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<td>• estimate, measure, label and compare using non-standard units of measurement: length, mass, time and temperature</td>
<td>How do we know whether or not something is likely to happen? Why are some things more likely to happen than others? How can we decide? How can we describe and compare the likelihood of something happening?</td>
<td>Can it happen? Students discuss situations, from stories read in class, in terms of the likelihood of their happening in real life: children can fly, dreaming about flying. Students label these situations as impossible to happen, unlikely to happen, likely to happen or certain to happen in real life. Students defend their answers. Comparing events that happen in the real world provides opportunities for students to clarify their ideas.</td>
<td>Students are given a list, or pictures, of situations and can place them in order of their likelihood, from impossible to certain. They can explain and defend their thinking.</td>
<td>Situations that come up naturally in the classroom, often through literature, present opportunities for discussing probability. Discussions need to take place in which students can share their sense of likelihood in terms that are useful to them. These discussions need to occur repeatedly. Our sense of probability often defies mathematical definitions and should not be dismissed. A good example is flipping a coin. Although every time you flip a coin the mathematical chance that it will fall on one side rather than the other is equal, many people would say that, after a run of one side, the chance of flipping the side we have not yet seen increases. There is some common sense in this.</td>
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<td>When would we want to measure something?</td>
<td>Measuring using non-standard units</td>
<td>Students can reasonably estimate the length, mass and/or temperature of given items using non-standard units. They can explain how they determined their estimates and how they can check them. Students can check their estimations by actually measuring the objects. Based on their findings, they can estimate the length, mass and/or temperature of other items and explain how they determined their estimates.</td>
<td>Students can measure circumference with string and measure the weight, in non-standard units, using a balance scale. Students can compare and order the parcels. With practice, their accuracy and speed will increase.</td>
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<td>Situations that come up naturally in the classroom, often through literature, present opportunities for discussing probability. Discussions need to take place in which students can share their sense of likelihood in terms that are useful to them. These discussions need to occur repeatedly. Our sense of probability often defies mathematical definitions and should not be dismissed. A good example is flipping a coin. Although every time you flip a coin the mathematical chance that it will fall on one side rather than the other is equal, many people would say that, after a run of one side, the chance of flipping the side we have not yet seen increases. There is some common sense in this.</td>
<td>The process of measuring is identical for any attribute: choose the unit, compare that unit to the object and report the number of units. Students develop an understanding of measurement by using materials from their immediate environment as measuring units such as corks, beads and beans. They begin to investigate how units are used for measurement and how measurements vary depending on the unit that is used. Students will refine their estimation and measurement skills by basing estimations on prior knowledge, measuring the object and comparing the actual measurements to their estimations. Developing personal referents will help in this process. Experiences of filling containers with sand and water will eventually lead students to an understanding of capacity/volume. This, however, is not a specific expectation at this time.</td>
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<td>Measurement (cont.)</td>
<td>Students will:</td>
<td>Why do we use standard units?</td>
<td>Students can explain why two students may get different answers when they measure the same space in the classroom with their feet.</td>
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<td>• use a calendar to determine the date, and to identify and sequence days of the week and months of the year</td>
<td>Why do we use calendars?</td>
<td>Students measure, label and compare the results of measuring the same object using different non-standard units such as their hands and pencils. Students explain why the measurement may change even though the object remains the same.</td>
<td>Informal and personal systems of measurement need to become standardized so we can compare measurements accurately and communicate measurement in a consistent way. It is important to know how accurate a measure needs to be or can ever be.</td>
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<td></td>
<td>How can we use a calendar to find out the date?</td>
<td>Students make their own tools for measuring.</td>
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<td></td>
<td>What patterns are there on the calendar?</td>
<td>Students can compare their estimates to the actual number of times they wrote their name.</td>
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<td></td>
<td>form</td>
<td>Students need to understand the duration of time (3:12pm).</td>
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<td></td>
<td>Why is it important to be able to put the days of the week and the months of the year in the correct order?</td>
<td>Students can compare their estimates with the actual number of times they wrote their name.</td>
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<td></td>
<td>responsibility</td>
<td>Students need to understand how long a minute is.</td>
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<td></td>
<td>• estimate, identify and compare lengths of time: second, minute, hour, day, week, month</td>
<td>How long will it take?</td>
<td>Students can compare their estimates with the actual number of times the timer was set.</td>
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<td></td>
<td>What language do we use to describe time?</td>
<td>Students set and reset a timer for 30 minute intervals, keeping a record of every time the timer goes off until lunchtime.</td>
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<td></td>
<td>form</td>
<td>Students predict how long they think it will be until lunchtime, based on how many half hour intervals will occur during this time.</td>
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<td></td>
<td>Why would we want to measure time?</td>
<td>Students can compare their estimates to the actual number of times the estimate that elapsed. Students predict the time left until the end of the school day. They can justify why their estimate for the time remaining in the school day is more or less than previous estimates.</td>
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<td></td>
<td>function</td>
<td>Students can predict how long they think it will be until lunchtime, based on how many half hour intervals will occur during this time.</td>
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<td></td>
<td>What can you do in a second/a minute/an hour?</td>
<td>Students need to understand time in terms of the duration of time (an hour) and the specific time (3:12pm). These are two very different notions. Daily calendar activities and classroom schedules will help develop these concepts.</td>
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<td></td>
<td>How do we know when a day/week/month begins and ends?</td>
<td>Repeat these activities several times over a few days until students have an understanding of how long a minute is.</td>
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<td>• read and write the time to the hour, half hour and quarter hour.</td>
<td>Students are given a specific time and can write what the time will be 15 minutes from then, an hour from then and a half hour from then.</td>
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<td></td>
<td>How can counting by 5 and 10 help us to tell the time?</td>
<td>Being able to tell the time is a concept that is often pushed too early. Students need to understand the duration of time before being introduced to the moment of time.</td>
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<td></td>
<td>How can knowing about fractions help us to tell the time</td>
<td>Students use digital and analogue clocks to tell time to the quarter hour, half hour and hour.</td>
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<tr>
<td></td>
<td>Why is time referred to in quarters and half hours?</td>
<td>Students are given a specific time and can write what the time will be 15 minutes from then, an hour from then and a half hour from then.</td>
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<tr>
<td></td>
<td>How do we record the time of day?</td>
<td>Students need to understand the duration of time before being introduced to the moment of time.</td>
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</tbody>
</table>
### Shape and space

**Students need to understand the interrelationships of shapes, and the effects of changes to shape, in order to understand, appreciate, interpret and modify our two-dimensional and three-dimensional world.**

**Students will:**

- use what they know about 3-D shapes to see and describe 2-D shapes

#### Specific expectations

The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

#### Sample questions

Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

- How can knowing about 2-D shapes help you to learn more about 3-D shapes?
- How are these shapes the same as, and different from, each other?
- Which 2-D shapes make up this 3-D shape?
- Can you explain how the cereal box was made? change

#### Sample activities

- Finding 2-D faces on 3-D shapes
  - Students trace around, paint and print from the faces of 3-D shapes.
  - Students deconstruct 3-D shapes into 2-D faces.
  - Students cut up a 3-D container in order to see the 2-D elements. They sort and mark the 2-D pieces according to shape. For example, colour all of the squares red and all of the rectangles blue. Then they reconstruct the container and describe the 2-D aspects of it.

- Sorting 2-D and 3-D shapes
  - Students sort 2-D and 3-D shapes (real objects and geometric shapes) into different sets. They discuss what is the same/different about the shapes and describe the properties.
  - Students explain why they sorted the shapes in the way that they did and provide labels for each set based on their explanation.
  - Students look for examples of shapes in the immediate environment and explain how they recognize them by their characteristics.

- Creating 2-D shapes
  - Students are given a description of a 2-D shape and are asked to create the shape using different materials: paper, pencil, tape, paint, geoboards.

#### Sample assessments

- Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.
  - When given a 3-D shape, students can describe the 2-D elements of it. They can create 2-D shapes and assemble them to create a 3-D shape.
  - Students can draw a 2-D and 3-D representation of a 3-D shape.
  - Students can describe how their 2-D drawing helped them construct their 3-D drawing.

#### Resources and comments

- **2-D shapes**, in reality, only exist as faces of 3-D objects. We help students understand the concept of planes by teaching 3-D before 2-D.

Computer programs that create 2-D and 3-D shapes would be excellent tools to use for this unit of study. This takes away the pressure of students having to draw the objects themselves while building on spatial awareness and 2-D and 3-D properties.

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Computer programs that create 2-D and 3-D shapes would be excellent tools to use for this unit of study. This takes away the pressure of students having to draw the objects themselves while building on spatial awareness and 2-D and 3-D properties.

- **Sample activities**
  - Given a description of a 2-D or 3-D shape, students can create it and name it.
  - Students can describe the properties of a pre-organized set of shapes.
  - Students can determine the attributes by which a set was sorted and can label the set accordingly. They can find other shapes to add to the collection.
  - Students can sort shapes into many different sets. Each time they can explain how they sorted each set and can provide an appropriate label.
  - Students can point out a variety of 2-D and 3-D shapes in the immediate environment and describe the attributes of each shape.
  - Students can use the correct mathematical vocabulary to describe shapes.

- **Resources and comments**
  - There must be a wide variety of materials available for students to construct 3-D shapes from 2-D faces.
  - Through creating and manipulating shapes, students align their natural vocabulary with more formal mathematical vocabulary and they begin to appreciate the need for this precision.
  - Students need to understand the properties of 2-D and 3-D shapes before the mathematical vocabulary associated with shapes makes sense to them.

- **Notes for teachers**
  - Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.
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<tr>
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<td>Students will:</td>
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<td><strong>Finding symmetry</strong></td>
<td>Students look for examples of symmetry in magazines and architecture. Advertising logos and car wheels are often good examples of symmetrical designs. Students collect and discuss the examples they find.</td>
<td><strong>Sample activities</strong></td>
<td>Students can find examples of symmetry in their immediate environment. They can explain what makes something symmetrical and what would cause them to be asymmetrical.</td>
<td><strong>Resources and comments</strong></td>
</tr>
<tr>
<td>• find and explain symmetry in their immediate environment</td>
<td></td>
<td><strong>Creating symmetrical designs</strong></td>
<td>With a partner, students use cubes and grid paper to create a symmetrical design. They fold the grid paper in half to create a line of symmetry. One student places a cube in a square on the grid, the other student has to place a cube in a square on the other side of the line of symmetry to make the design symmetrical. Students use mirrors to verify that their design is symmetrical.</td>
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<tr>
<td>• create and explain simple symmetrical designs</td>
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<td><strong>Following directions</strong></td>
<td>Students create a maze in the playground or by moving the furniture around in the classroom. They are paired up with another student and take turns blindfolding each other. They give specific directions to their blindfolded partner in order to move them through the maze.</td>
<td></td>
<td></td>
<td>Many people confuse their right from their left. Students need many opportunities to experience and quantify space in a direct kinaesthetic manner.</td>
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<tr>
<td>The regions, paths and boundaries of natural space can be described by shape.</td>
<td></td>
<td>• give and follow simple directions, describing paths, regions and boundaries of their immediate environment and their position: left, right, forward and backward.</td>
<td>When would we need to give/follow directions to get somewhere? How can you describe your position to somebody who cannot see you? How precise do directions need to be?</td>
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<td><strong>Pattern and function</strong></td>
<td>To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called functions. This builds a foundation for the later study of algebra.</td>
<td><strong>Specific expectations</strong></td>
<td></td>
<td><strong>Sample questions</strong></td>
<td>Where do we find patterns? What do you know about patterns? How can patterns help us? In what ways are these patterns similar and/or different? What can go in the missing space? Is that the only possible cube/colour? In how many different ways can you describe this pattern?</td>
<td><strong>Sample activities</strong></td>
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<td>• create, describe and extend patterns</td>
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<td><strong>Creating and extending patterns</strong></td>
<td>Students are given a pattern created with pattern blocks. They are asked to describe the pattern and to extend it. Using coloured cubes, or squares, the teacher makes a pattern and leaves spaces for the student to complete. Students prepare patterns for each other to complete. Later, substitute a cloud shape or box in the space to be filled. Using a variety of manipulatives, students create and describe two different patterns. They compare whether their two patterns are the same and justify their answer.</td>
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### Content | Specific expectations | Sample questions | Sample activities | Sample assessments | Notes for teachers
---|---|---|---|---|---
**Pattern and function (cont.)** Students will:  • recognize, describe and extend patterns in numbers: odd and even, skip counting, 2s, 5s and 10s  - What patterns do you notice in the numbers you know?  - How can we know what number comes next in a sequence?  - How can we work with number patterns to help us?    **Patterns in numbers** Students model a number pattern using manipulatives. (They have a group of 2 cubes, then a group of 4, then 6, to represent counting by 2.) They are asked to describe, explain and extend another student’s number pattern. This can be done orally, with the use of manipulatives, and/or in written form through pictures, equations or colouring in the pattern on a 100s chart.  - Using a number line, students mark the various patterns of skip counting. The 2s can be underlined, the 3s put into triangles and the 4s put into squares.  - Given a number line, students mark the next five numbers in this sequence. They can explain how they know this.  - Students can describe, explain and extend any pattern they find in the 100’s chart.  - Students turn their patterning skills to the numbers they already know. The patterns they find will help to deepen their number concepts and to extend their range of numbers.  • identify patterns and rules for addition: $4 + 3 = 7$, $3 + 4 = 7$ (commutative property)  - How are $4 + 3$ and $3 + 4$ connected? connection  - In what ways are $4 + 3$ and $3 + 4$ the same?  - In what ways are $4 + 3$ and $3 + 4$ different?  - Why does the sum remain the same even if the addends appear in a different order? causation  - Why do these calculations produce patterns? causation    **Patterns and rules for addition** Students model $5 + 2$ and $2 + 5$. They use their models to compare and contrast the two equations.  - Students can make a model to show how $4 + 3$ and $3 + 4$ are connected. They can explain the patterns and rules used.  - Many students intuitively work out the commutative property. It is important that they understand that, while it doesn’t matter in which order sets are added as the answer remains the same, $4 + 3$ is generated by a different story to $3 + 4$.  • identify patterns and rules for subtraction: $7 - 3 = 4$, $7 - 4 = 3$  - How are $7 - 3$ and $7 - 4$ connected?  - In what ways are $7 - 3$ and $7 - 4$ the same?  - In what ways are $7 - 3$ and $7 - 4$ different?  - Why is the order of numbers in a subtraction equation so important?  - What effect does the order of numbers have on the difference?  - Students can make a model to show how $7 - 3$ and $7 - 4$ are connected. They can explain the patterns and rules used.  - It is important for students to explore the patterns and rules for addition and subtraction through problem solving. Real-life contexts should be used as often as possible.  • model with manipulatives, the relationship between addition and subtraction: $3 + 4 = 7$, $7 - 3 = 4$.  - How are addition and subtraction connected? connection  - What do you know about addition that can help you with subtraction?  - How can knowing your addition facts help you with your subtraction facts?  - Students are given the numbers six, two and eight and are asked to model as many addition and subtraction equations as they can, using all three numbers. Students model and write the equations. They determine and explain the connection between addition and subtraction.  - Students can make a model to show how $3 + 4$ and $7 - 3$ are connected. They can explain the patterns and rules used.  - Besides introducing the important mathematical concept of inverse operations in a user-friendly context, finding “families” of number facts is another way for students to learn and remember the subtraction facts.

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**Subject: mathematics**  
**Age range: 5–7 years**  
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### Subject: Mathematics

#### Age range: 5–7 years

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#### Number

Our number system is a language for describing quantities and relationships between quantities. The value attributed to a digit depends on its place within the base 10 system. The operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends upon how the result will be used.

The availability of computers and calculators has provided us with an unprecedented opportunity to explore relationships and rules in the number system. Students now have the opportunity to use technology to find patterns, explore relationships, and develop algorithms that are practical for them. The educational experiences of students must include the use of technology.

Students will:
- Read, write and model numbers, using the base 10 system, to 100.
- Count (in 1s, 2s, 5s and 10s), compare and order numbers to 100.
- Estimate quantities to 100.

**Introducing the base 10 system**

Students use manipulatives that can be physically put together and taken apart, such as linking cubes. Once students have understood this concept, more abstract manipulatives, such as base 10 blocks, can be used.

Students are shown a set of objects and are asked to write the numeral indicating the quantity. Students are given a number and asked to model the quantity by using manipulatives or by drawing pictures.

Use number-name and numeral cards to label sets of objects.

**Counting patterns**

Students use a calculator to find number patterns. They start with zero, add 2, and record the sum on a 100’s chart. They continue to add 2 to each sum until they see a pattern forming. They repeat the activity by starting with zero and adding 5 each time, as well as starting with zero and adding 10 each time.

When given a larger group of objects to count, students are encouraged to group them in 2s, 5s or 10s to help them count accurately.

**Estimating to 100**

Students are shown a set of 10 cubes. They are then shown another set of 40 cubes. They are told how many are in each set. They are shown another set of 20 cubes and are asked to estimate how many cubes are in this set. Students are asked to explain the strategies they used and how knowing the amounts contained in the other two sets helped them with their estimation.

Students are shown three glass jars of the same size and shape. Each jar contains cubes of the same size. One jar has 30 cubes, one has 60 and the other has 90. Students can estimate how many cubes are in each of the jars and can explain the strategies they used to do this.

In English, the numerals 10 to 19 are harder to encode and decode than 20 to 99 since the language is less regular. 24 is written in the same order as it is said whereas with 17, the second digit is heard first. There is also confusion between the teens and the tens.

Students need to use manipulatives and patterns of number to understand the concept of the base 10 system.

Students need practice in using manipulatives to represent a certain number and writing numbers that have been represented with manipulatives.

Students need to practise reading, writing, comparing and ordering numbers in a variety of real-life situations.

Children’s literature also provides rich opportunities for developing number concepts.

Students need to develop an understanding of our number system and the patterns that arise within it, including the relationship between numbers, as well as the properties of numbers. Students need to use numbers in many situations in order to apply their understanding to new situations.

Students need to practise comparing and ordering numbers in real-life situations. This is a great opportunity to teach the neglected concept of difference.

Students need a lot of practice in order to develop their estimation skills.

They must also have the opportunity to check their estimates and to refine them.

Exposure to various mathematical strategies will help students to refine their estimations.

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<td>Number (cont.) Students will:</td>
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<tr>
<td>• use mathematical vocabulary and symbols of addition and subtraction: add, subtract, difference, sum, ± =</td>
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<td>• read, write and model addition and subtraction to 20 (with and without regrouping)</td>
<td>How can we record our mathematical problems? How can we explain our mathematical thinking to others?</td>
<td>Add and subtract to 20 Students make up addition and subtraction word problems for a partner. Students play addition and subtraction games, such as Snap or Bingo, using different written and numerical forms.</td>
<td>Students are given two problems, one involving carrying, one involving decomposition. They can solve the problem using cubes and place-value mats. Students can record the equations that go with their models, and can explain how they solved them.</td>
<td>Students need to develop an understanding that mathematics is a language that is communicated through special mathematical vocabulary and symbols. Multiple definitions of subtraction need to be emphasized, especially “how many more”, as this leads to the mathematical concept of difference.</td>
</tr>
<tr>
<td>• automatically recall addition and subtraction facts to 10</td>
<td>How can we model addition and subtraction problems with larger numbers? What equation could help us to solve that problem? What problem could help us to solve that equation? How do we know when to regroup? reflection</td>
<td>Number facts to 10 Throw two dice. Add (or subtract) the two numbers, initially through counting but building up to automatic recall. Students are asked to solve the problem 4 + 3 in their head and to explain the strategy they used. They are then given the problems 4 + 5 and are asked to solve it using a different strategy. Students are asked which strategy was easiest to use and which allowed them to solve the problem the fastest. Students can explain the strategies they use to solve addition and subtraction problems in their head. Students can record their accuracy and speed of recalling addition and subtraction facts to 10. They are asked if they are satisfied with that speed and what they could do to become faster. Students keep a record of their speed and accuracy over time to determine improvement.</td>
<td>Students need to practise their addition and subtraction facts through a variety of games, and individual or group activities, that promote automaticity. They need to explore several methods and discuss these with their peers. To be useful, addition and subtraction facts need to be recalled automatically. Research clearly indicates that there are more effective ways to do this than “drill and kill”. Above all, it helps to have strategies for working them out. Counting on, using doubles and using 10s are good strategies, although students frequently invent methods that work equally well for themselves.</td>
<td></td>
</tr>
<tr>
<td>• describe the meaning and use of addition and subtraction</td>
<td>How, and in what ways, do numbers change when we add/subtract?</td>
<td>When do we add or subtract? Prepare simple addition and subtraction problems with the mathematical symbols missing. Students are detectives working out which symbol is missing and explaining how they know this.</td>
<td>Students can describe what will happen in an addition or subtraction equation. They can explain when they would use an addition or subtraction equation.</td>
<td></td>
</tr>
</tbody>
</table>
### Number (cont.)

Students will:

- **explore and model multiplication and division** using their own language/methods

**Specific expectations**

The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

**Sample questions**

Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

**Sample activities**

All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen).

**Sample assessments**

Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.

#### Exploring multiplication and division

Solve real-life problems involving multiplication and division in groups: “There are 20 cubes for this activity and everyone needs to have the same number of cubes.” “There are 5 people in this group. How many cubes will each person get?” Students work with their group to solve the problem. Each group shares their strategy and mathematical thinking with the other groups.

Students can **model** the problem and explain the strategy they used, eg everyone in the group needs to have three pieces of paper for this activity. There are four people in the group. How many pieces of paper will your group need?

Students, at this stage of development, need to develop a sense of multiplication and division as repeated addition and subtraction. While they are just beginning to develop a sense of multiplication and division through the use of manipulative materials and real-life situations, they are not developmentally ready to use the mathematical language or symbols of these operations.

#### Halves and quarters

Students cut wholes into parts to understand their relationships and to practise using the language to describe the parts.

Eight students come to the front of the room. The class looks for different ways to sort these students into groups.

If something is cut in half or in quarters, students can explain how many parts it has and how they know this.

Students look at a picture and can tell the teacher into how many parts it was cut or shared.

The class can answer questions about these groupings: What fraction of the group is male? What fraction of the group is female? What fraction of the group is wearing red? What fraction of the group has curly hair?

It is important to include situations where the **whole** is a set of objects, as well as one object that is cut into parts.

Initial instruction needs to emphasize oral language only and connect it to the **models**.

#### Could that be right?

Students share their ideas, have a chance to work them out and check for reasonableness.

Students can use their knowledge of numbers to estimate an answer.

They can check their answer against the estimate and comment on its reasonableness.

#### Choosing a sensible method

Students are given a problem and are asked to solve it using two different strategies.

Students are asked to explain which strategies were used and which was the most effective.

Students can describe two strategies for solving addition and subtraction problems.

Students can explain the strategy they used, the reason for choosing that particular strategy, and the advantages and disadvantages of each.

Students need to be exposed to a variety of strategies that will enable them to construct meaning about mathematical operations. They need to explore various methods of computation and discuss them with their peers.

Students need to have strategies that make sense to themselves. These may be their own “invented” strategies or those that they have adopted.

---

Students will:

- **use fraction names (half, quarter) to describe part and whole relationships**

- **estimate the reasonableness of answers**

- **select and explain an appropriate method for solving a problem.**
### Overall expectations

Students will have the opportunity to identify and reflect upon “big ideas” by making connections between the questions asked and the concepts that drive the inquiry. They will become aware of the relevance these concepts have to all of their learning.

### Data handling: statistics and probability

Students will discuss, compare and create sets that have subsets; design a survey; and process and interpret the data on a bar graph where the scale represents larger quantities. They will manipulate information in a database. They will find, describe and explain the mode in a set of data and will use probability to determine the outcome of mathematically fair and unfair games.

### Measurement

Students will estimate, measure, label and compare length, mass, time and temperature using formal methods and standard units of measurement. They will determine appropriate tools and units of measurement including the use of small units of measurement for precision (cm, mm, °C). They will also estimate, measure, label and compare perimeter and area, using non-standard units of measurement. Students will model the addition and subtraction of money and be able to read and write time to the minute and second.

### Shape and space

Students will sort, describe and model regular and irregular polygons, including identifying congruency in 2-D shapes. They will combine and transfer 2-D shapes to create another shape. They will identify lines and axes of reflective and rotational symmetry, understand angles as a measure of rotation and locate features on a grid using coordinates.

### Pattern and function

Students will recognize, describe and analyse patterns in number systems. They will identify patterns and rules for multiplication and division, together with their relationship with addition and subtraction. They will model multiplication as an array and use number patterns to solve problems.

### Number

Students will read, write, estimate, count, compare and order numbers to 1000, extending understanding of the base 10 system to the thousands. They will read, write and model multiplication and division problems. They will use and describe multiple strategies to solve addition, subtraction, multiplication and division problems, reasonably estimating the answers. They will compare fractions using manipulatives, mathematical vocabulary and fractional notation. They will understand and model the concept of equivalence to one.

* See glossary for explanation of italicized terms.

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<td>Discussing and creating sets</td>
<td>All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the FDP happen).</td>
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<td>* discuss, compare and create sets from data that has subsets using tree, Carroll, Venn and other diagrams*</td>
<td>Students sort drawings or names of objects, rather than the objects themselves, using a tree diagram.</td>
<td>Given a specific set, such as names in the class, students suggest labels for paths/branches of a tree diagram.</td>
<td>Students can draw a diagram to show the names of known coins and whether they are copper- or silver-coloured. Students can draw a Venn diagram to show the sets “students in our school”, “students in our class” and “students in our group.”</td>
<td>Natural examples of subsets can be explored within the units of inquiry.</td>
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<td></td>
<td>Students investigate how many different combinations can be created from a set of leisure clothes. The concept of a tree diagram is introduced and is used to show a systematic way of accounting for all possibilities.</td>
<td>Students can highlight areas where the information corresponds in the two formats.</td>
<td>A great deal of practice may be necessary for some students to describe the two (or more) attributes often shown in Carroll and Venn diagrams.</td>
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<td></td>
<td>Use the same information (teacher-generated and given to the students or collected by the students) and display it on a Carroll diagram and a Venn diagram.</td>
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<td>Presenting data diagrammatically often helps understanding or analysis and facilitates explaining data to others.</td>
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<td></td>
<td>When and why would we use these diagrams?</td>
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<td>It is important to remember that the chosen format should illustrate the information without bias.</td>
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<tr>
<td><strong>Data can be recorded, organized, displayed and understood in a variety of ways to highlight similarities, differences and trends.</strong></td>
<td><strong>Sample questions</strong></td>
<td><strong>Sample activities</strong></td>
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<td><strong>What is the relationship between these two sets?</strong></td>
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<td><strong>What smaller set does this set contain?</strong></td>
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<td><strong>When and why would we use these diagrams?</strong></td>
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<td><strong>How can we find out about this?</strong></td>
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<td><strong>What are the most important pieces of information we need?</strong></td>
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<td><strong>In what ways can we display the data collected?</strong></td>
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<td><strong>How can we interpret the data?</strong></td>
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<td><strong>Who might be interested in, or be able to use, the results of our survey?</strong></td>
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<td><strong>What could you do differently if you repeated the survey? reflection</strong></td>
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<td><strong>Designing a survey</strong></td>
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<tr>
<td><strong>Students choose a topic and frame questions appropriate for a small survey. Before conducting the survey, they discuss the best way to obtain the information: who/how many people will be asked, clear wording for questions, recording methods. They conduct the survey, collate and represent the data using a variety of forms, and discuss the findings.</strong></td>
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<tr>
<td><strong>Students can carry out a survey to collect and collate information about classmates.</strong></td>
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<tr>
<td><strong>Students can represent the data and are able to explain any results or patterns.</strong></td>
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<td><strong>Presenting data diagrammatically often helps understanding or analysis and facilitates explaining data to others.</strong></td>
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</table>
### Subject: Mathematics  
**Age range:** 7–9 years

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<td><strong>Data handling: statistics (cont.)</strong></td>
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<tr>
<td><strong>Specific expectations</strong></td>
<td>- collect and display data in a bar graph and interpret results</td>
<td>- use the scale on the vertical axis of a bar graph to represent large quantities</td>
<td>- find, describe and explain the mode in a set of data and its use</td>
<td>- understand the purpose of a database by manipulating the data to answer questions and solve problems.</td>
</tr>
</tbody>
</table>
| **Sample questions** | - What are the features of a bar graph?  
- What does this bar graph tell us? function  
- How do these features compare with other graphs you have made?  
- How is a bar graph like a pictograph?  
- How are they different? | - Making the data fit the graph  
- Use scales where each unit is not recorded but marks are indicated in intervals of 2 or 5. Use squared paper to give easily marked intervals.  
- When a frequency table is being prepared, introduce students to a tally system to make the total easier to count. | - Making a bar graph  
- Teachers could illustrate to students by using Cuisenaire rods and then exchanging these for the matching rod representing the total number in the set: a yellow rod would replace five white unit rods.  
- Students name the TV programme they watch at a given time. Data is represented using interlocking blocks. Each column is then labelled. The data could be transferred to paper. | |
| **Sample activities** | - Bar graphs  
Teachers could illustrate to students by using Cuisenaire rods and then exchanging these for the matching rod representing the total number in the set: a yellow rod would replace five white unit rods.  
Students name the TV programme they watch at a given time. Data is represented using interlocking blocks. Each column is then labelled. The data could be transferred to paper. | - What is the mode?  
Surveys are carried out to find the mode for the class/school of height, weight, amount of allowance, number of siblings, favourite cartoon, favourite singer/band, favourite sports team, nationalities, ages.  
Using the mode as a guide, students produce a shirt that will fit most of the class, and make a paper pattern for it. | - When given a set of data, students can determine the mode and explain why knowing the mode is useful in this situation.  
Students can describe in their own words what the mode value is (oral).  
They can find the mode for the number of sweets in several bags (practical). | |
| **Sample assessments** | - Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways. | - Students can describe and discuss key features of the graph.  
Students can develop questions they want to answer given a particular database.  
This demonstrates knowledge of fields. They can use the database to answer their questions and present findings to the class. | - Situations that come up naturally in the classroom or form part of the units of inquiry present opportunities for students to determine a scale. Students need to be able to suggest possible solutions, put them forward for discussion, and defend or adapt their choice. | |
| **Resources and comments** | - Students will collect and display data in a bar graph and interpret results | - Making the data fit the graph  
Use scales where each unit is not recorded but marks are indicated in intervals of 2 or 5. Use squared paper to give easily marked intervals.  
When a frequency table is being prepared, introduce students to a tally system to make the total easier to count. | - Using data that has been collected and saved is a simple way to begin discussing the mode. A further extension of mode is to formulate theories about why a certain choice is the mode. | |
| **Notes for teachers** | - At this stage of development, students can begin to use bar graphs to record data because they can be simply viewed as putting the squares or tiles together in a rectangle. They should be given opportunities to do this almost on a daily basis. The units of inquiry will also be rich with opportunities for graphing. | - Making a bar graph  
Teachers could illustrate to students by using Cuisenaire rods and then exchanging these for the matching rod representing the total number in the set: a yellow rod would replace five white unit rods.  
Students name the TV programme they watch at a given time. Data is represented using interlocking blocks. Each column is then labelled. The data could be transferred to paper. | - Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners. |
### Data handling; probability

There are ways of finding out if some outcomes are more likely than others. Probability can be expressed qualitatively by using terms such as unlikely, certain or impossible. It can be expressed quantitatively on a numerical scale.

The availability of computers and calculators has provided us with an unprecedented opportunity to process data and explore probability in more thoughtful, efficient and imaginative ways. The educational experiences of students must include the use of technology.

Students will:
- use probability to determine mathematically fair and unfair games and to explain possible outcomes.
- have we tested enough? responsibility
- what makes a game fair? is it fair?
- have we tested enough? responsibility

### Measurement

To measure is to attach a number to a quantity using a chosen unit. However, since the attributes being measured are continuous, ways must be found to deal with quantities that fall between the numbers. It is important to know how accurate a measure needs to be or can ever be.

- estimate, measure, label and compare using formal methods and standard units of measurement: length, mass, time and temperature
- how can we measure the size?
- how can we estimate the amount of space it takes up?
- which of these rocks is the heaviest?
- measuring using standard units

### Notes for teachers

Students can invent a game. They can design two sets of rules, one that is fair and another that is unfair. They can explain why this is the case.

Students can count how many ways there are of making each sum and can explain if this game is fair.

Students can name possible outcomes (two heads and a tail, three heads) when three coins are tossed at the same time. They can repeat the experiment to show that some outcomes are much less likely than others.

Students can discuss the possible totals. They can match predictions against actual outcomes.

Students discuss the range of acceptable estimates, and how they determined their own estimate, helps everyone develop a strategy that makes sense and is useful.
### Measurement (cont.)

#### Specific expectations

- Students will:
  - select appropriate tools and units of measurement
  - describe measures that fall between numbers on a measure scale: 3½kg, between 4cm and 5cm
  - estimate, measure, label and compare perimeter and area

#### Sample questions

- What do we do when the object we are measuring falls between two whole numbers?
- How can we describe a measure that falls between whole numbers on the scale?
- Which parts of the tools help us?
- What do we call the units between the numbers on the scale?

#### Sample activities

- Measures that are not exact
  - From a supermarket catalogue, students select up to 10 objects whose mass is given in grams and kilograms. They sort the items into categories, eg those within 100g of 1kg, and those that are between 800g and 1.5kg.
  - Students collect maximum and minimum daily temperatures from newspapers, or the web, for capital or regional cities around their home country. Students look for patterns over time.

- What is perimeter? form
  - What is area?
  - How is area connected to perimeter? connection
  - What happens when we change from one unit to another? change
  - What did we use to measure with and why did we choose it?

- Looking at area and perimeter
  - Students are asked to choose an object that is between 5cm and 6cm long. “How we can describe the size?”

#### Sample assessments

- How can we measure this?
  - Students measure a variety of objects: short and long lengths, light and heavy masses, hot and cold temperatures and so on. Before measuring, they must decide upon the attribute they wish to measure. They select the tools they think will best measure that attribute. After measuring, students discuss whether another tool, or choice of unit, would have worked better.
  - Give the students opportunities to measure the same attribute using units of different sizes (mm and cm). This will help them to select the most appropriate unit.
  - Students estimate, in centimetres, the lengths/widths of their index fingers/feet. Exact measurements are then taken and results checked.
  - Students research a sport of interest to them and discuss the importance of precise measurements in that sport (eg Olympic records).
  - Students can match each card with the object measured.

- How will we know what students have learned?
  - Students select an appropriate measuring tool from a variety available (ruler, tape measure, metre rule, trundle wheel) to measure the length of a book.
  - Students, individually or in pairs, can choose the most appropriate unit for measuring the length, height or width of a designated object from a list.
  - Each student measures, in centimetres, a feature (height, width, length) of a portable object in the classroom. Results are recorded on cards. Students can match each card with the object measured.

- Resources and comments
  - A wide range of measuring tools should be available to the students: 30cm rulers, metre sticks, 30m tapes, trundle wheels, height measurement charts, tape measures, bathroom scales, pan balances, kitchen scales, masses (1kg, 500g, 100g), sand timers, egg timers, analogue clocks, digital clocks, stopwatches, calendars.
  - Teachers should find ways to ensure that learners understand the tasks and expectations. Manipulatives facilitate understanding for all learners.

### Notes for teachers

- Teachers should find ways to ensure that learners understand the tasks and expectations.
- Manipulatives facilitate understanding for all learners.
### Measurement (cont.)

**Specific expectations**

The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

**Sample questions**

- Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding.
- All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

**Sample activities**

**Looking at area and perimeter (cont.)**

- Students use non-standard units of measurement to measure and compare two regular or irregular shapes. Students discuss which shape has the larger area and how this relates to the unit chosen. 
- Standard units of measurement can then be introduced.
- Students use square tiles to make shapes with the same area but different perimeters.
- Students use geoboards to investigate how different shapes can have the same area.

**Addition and subtraction with money**

- Students play trading games, using play money, with an emphasis on the base 10 system.
- Each student is allowed a set amount of money (50 dollars, pounds, euros etc.) They buy items from a catalogue and keep a running total. They try to come as close as possible to spending the money without going over.

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<td>Are our estimates realistic?</td>
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<td><strong>model addition and subtraction using money</strong></td>
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<td><strong>read and the time to the minute and second, using intervals of 10 minutes, 5 minutes and 1 minute, on 12-hour and 24-hour clocks.</strong></td>
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<td>• identify, describe and model congruency in 2-D shapes</td>
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<td>• combine and transform 2-D shapes to make another shape</td>
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**Shape and space** The regions, paths and boundaries of natural space can be described by shape. Students need to understand the interrelationships of shapes, and the effects of changes to shape, in order to understand, appreciate, interpret and modify our two-dimensional and three-dimensional world. Students need to have many experiences with shapes that are congruent. They need to practise describing what makes them congruent. Through modelling and manipulating shapes, students align their natural vocabulary of shape with more formal mathematical vocabulary and begin to appreciate the need for this precision.
### Shape and space (cont.)

**Specific expectations**

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**Resources and comments**

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**Students will:**

- create symmetrical patterns, including tessellation

**Creating symmetrical patterns**

Create a school logo using rotational symmetry.

- Tilling patterns.
- Describe the pattern of tiles on the classroom floor.
- Using only one kind of block, find which can be used to tile or cover an A5 size piece of paper. Then, using only two or three different blocks, try to create a tile pattern on the paper.

When students are shown a tiling design they can determine where flips, slides and turns are used in the design.

- Students can make a template of a tessellating shape. They can use this to create a pattern and to write a description of the pattern.

**Strategies for facilitating understanding:**

- In what ways are the patterns similar/different?
- Where in the designs do we see a flip (reflection), turn (rotation), slide (translations)?
- What makes certain designs work?
- What can you say about the area covered by each of our tiling patterns?
- Where else have you seen patterns like the tiling patterns?
- Does a…tessellate? function
- Do all quadrilaterals tessellate? Why? causation

---

**Identify lines and axes of reflective and rotational symmetry**

What is an axis of symmetry? form

- How does it help us define if things are symmetrical?
- What happens if you place the edge of a shape against the edge of a mirror?
- What happens if we move the mirror further and further away from the edge of a shape?
- Which of these shapes do you think you could fold so that the two sections match up?
- What happens if you place the mirror on the fold line?

**Identifying lines of symmetry**

Students take a survey and make a graph to show the most common shape in the classroom and to identify objects in the classroom that have reflective symmetry.

- Students use one 2-D shape to make three patterns: one by flipping, one by turning, and one by sliding the shape.
- Use geoboards to show shapes that have symmetry on each side of an imaginary mirror line and those shapes that do not.

**Strategies for facilitating understanding:**

- Students can find lines and axes of symmetry and explain how they know this.
- Students can sort a range of regular and irregular 2-D shapes into those with symmetry and those that are asymmetrical. They can draw the shapes showing any lines of symmetry.
- Students can write riddles describing the features of a secret shape.
- Students can make the mirror image or reflection of an object by placing the mirror line in several different positions.

**Angles as a measure of rotation**

How can we measure the size of a turn that we make?

- If we turn from north to south, how much of a turn do we make? North to east?
- How do mathematicians describe this?
- How can we compare and describe these rotations?

**Strategies for facilitating understanding:**

- Students devise and play a game that uses compass instructions and turns as part of the rules.
- Students practise moving themselves, robots and objects through space and keeping track of paths in order to describe them to someone else.
- Use a clock face, with moving hands, to ask the time if the minute hand makes a quarter turn. Ask students to justify their responses.

**Strategies for facilitating understanding:**

- Students can give three sets of directions to move an object from a starting position to a new position.
- When given a route map of a robot, ship or person’s movements, students can describe the visual image as a series of turns and can estimate the size of the turns.

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**Making the PYP happen**

- The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

**Sample questions**

Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

**Sample activities**

All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen).

**Sample assessments**

Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.

**Resources and comments**

Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.
### Subject: Mathematics  
**Age Range:** 7–9 Years  
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</table>
| **Shape and space (cont.)** | Students will:  
- locate features on a grid using coordinates. | **Locating features on a grid**  
Students put their desks in columns and rows to form a grid. They discuss ways to identify each desk by its position on the grid.  
Play a coordinate game using a 10 × 10 grid where the x’s and o’s are put on the line intersection instead of in spaces. They are placed according to their ordered pair names (eg 4, 2 where 4 refers to the horizontal axis and 2 refers to the vertical axis). The goal is to get 4 x’s or 4 o’s in a row.  
Students select a mystery object in class and describe its position accurately. Others guess the object. | **Locating features on a grid**  
Students put their desks in columns and rows to form a grid. They discuss ways to identify each desk by its position on the grid.  
Play a coordinate game using a 10 × 10 grid where the x’s and o’s are put on the line intersection instead of in spaces. They are placed according to their ordered pair names (eg 4, 2 where 4 refers to the horizontal axis and 2 refers to the vertical axis). The goal is to get 4 x’s or 4 o’s in a row.  
Students select a mystery object in class and describe its position accurately. Others guess the object. | **Sample assessments**  
Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways. |  
**Resources and comments**  
Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.  
A unit of inquiry in which students use maps is an excellent way to practise this skill and to explore different methods of arranging a grid. |

| **Pattern and function** | Students will:  
- analyse patterns in number systems to 100  
- recognize, describe and extend more complex patterns in numbers | **Analysing number patterns**  
Students use a 100s chart to code a variety of patterns: numbers that have 5 in them, numbers that end in zero. They then describe the patterns that emerge.  
Given a 100s chart with a missing number, students can correctly place the number and justify their answer. They can also describe and explain a pattern they find for themselves in the 100s chart. | **Analyzing number patterns**  
Students use a 100s chart to code a variety of patterns: numbers that have 5 in them, numbers that end in zero. They then describe the patterns that emerge.  
Given a 100s chart with a missing number, students can correctly place the number and justify their answer. They can also describe and explain a pattern they find for themselves in the 100s chart. | **Sample assessments**  
Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways. |

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Patterns are central to the understanding of all concepts in mathematics. They are the basis of how our number system is organized. Searching for, and identifying, patterns helps us to see relationships, make generalizations, and is a powerful strategy for problem solving.

**Functions** develop from the study of patterns and make it possible to predict in math problems. Use pattern blocks, attribute blocks, colour tiles, calculators, number charts, markers of all types (beans, buttons), spinners.
### Pattern and function (cont.)

**Specific expectations**
- Students will:
  - **understand and use the relationship between addition and subtraction:** 
    \[4 + 3 = 7, \ 7 - 3 = 4\]

**Sample questions**
- How are addition and subtraction connected?
- When might understanding this be helpful?
- How does the way in which numbers are grouped affect the answer?
- Patterns and rules for multiplication and division
  - Using a 100s chart, colour in every 3rd, 4th and 5th square to visually demonstrate patterns.
  - Students can predict the patterns or repeat patterns with a line to demonstrate patterns.

**Sample activities**
- Students can calculate the product of 6 \( \times \) 27 if the button is broken.

**Sample assessments**
- Students can explain how 5 + 3 = 8 and 8 - 3 = 5 are connected. They can make a model to show this.
- Suppose that this mathematical sentence represents a division fact:
  \[\frac{21}{3} = 7\]
  Students can write the other facts from this family.

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<td><strong>Sample questions</strong></td>
<td>How are addition and subtraction connected? When might understanding this be helpful? How does the way in which numbers are grouped affect the answer? Patterns and rules for multiplication and division</td>
<td>Using the relationship between addition and subtraction Using real-life problems, students experience the idea that if 3 apples + 4 apples = 7 apples, then if we have 7 apples and eat 3 of them, we will be left with 4 apples. Students can explain how 5 + 3 = 8 and 8 - 3 = 5 are connected. They can make a model to show this.</td>
<td>Students can explain how a multiplication or division pattern will continue, using a number chart or manipulatives. Besides introducing the important mathematical concept of inverse operations in a user-friendly context, finding families of number facts is another way for students to learn and remember the subtraction facts.</td>
<td>Commutative and associative properties apply to multiplication and addition but not to division or subtraction.</td>
</tr>
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<td><strong>Sample activities</strong></td>
<td>How are multiplication and division connected? Why do these calculations produce patterns? causation How does the way in which numbers are grouped affect the product? (commutative and associative property) Where else in mathematics does this rule apply? connection</td>
<td>How are multiplication and division related? Families of numbers are investigated to demonstrate the connection between multiplication and division: [3 \times 7 = 21] [7 \times 3 = 21] [21 + 3 = 7]</td>
<td>The PYP curriculum documents: mathematics © IBO 2003</td>
<td></td>
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<td><strong>Sample assessments</strong></td>
<td>How can our calculators help? How can our calculators be linked to a key concept. Some examples are noted below in bold.</td>
<td>How are multiplication and division related? Students work in pairs, using calculators to complete charts. They press the keys: <code>+ then 2 then =</code> One person presses then presses it again as they watch the pattern grow. Students are asked how they can use a calculator to find the product of 6 ( \times ) 27 if the button is broken.</td>
<td>Given a number to count by, students can predict the pattern of counting the first three entries. Use four-function calculators.</td>
<td></td>
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**Resources and comments**

- Students work in pairs with manipulatives, the relationship between multiplication and division
- Students can calculate the quotient of 96 \( \div \) 4 if the key is broken.
- Care should be taken to help students choose an appropriate starting number (dividend) and divisor to avoid remainders.
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<td>Students will:</td>
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<td>• model multiplication as an array</td>
<td>How can we model a multiplication equation? Which products can be represented as squares and which as rectangles?</td>
<td>Arrays: Students use arrays to explore multiplication facts and the relationship between various multiplication facts. Students make models of all the possible rectangles with an area of 1 to 25, using tiles. Students use counters to explore making square and rectangular numbers. Students create rectangular arrays that have equal areas but different dimensions. Progress from making arrays with tiles to using graph paper.</td>
<td>Students can model $2 \times 3$ and $3 \times 2$ and can explain how they are the same and how they are different. Students can explain that the area of a rectangle can be described as a multiplication problem. Students can describe all the possible dimensions of a regular-shaped garden if its area is 24 square units. They can calculate the area of a 8m by 6m garden lawn, using an array.</td>
<td>Besides deepening their understanding of multiplication, students will discover the commutative property of multiplication, which will help them in learning the multiplication facts. The resulting records of these rectangles form the basis of discussion about the square numbers, prime numbers, multiples and factors.</td>
</tr>
<tr>
<td>• understand and use number patterns to solve problems (missing numbers).</td>
<td>What symbols do mathematicians use to represent unknown numbers?</td>
<td>Missing numbers: Students practise with a function machine to understand that a function applies a consistent rule when it operates on a number. Students continue the pattern to build a row of 100 triangles: How many toothpicks are needed? 1 triangle needs 3 toothpicks, 2 triangles need 5 toothpicks, 3 triangles need ____ toothpicks. Try this with squares, pentagons, hexagons. Suppose everyone in the room shakes hands with every other person in the room. How many handshakes will that be?</td>
<td>Students can identify the rule or function being used in the following equation: $5 + ___ = 6$ and $7 + ___ = 8$. Students can identify missing elements of number patterns and can discuss attributes of each pattern.</td>
<td>Generalizing a pattern and calling it a function is a central idea in mathematics. Students need practice in the formation of functions and in determining functions from a t-chart (graph) in order to develop algebraic thinking.</td>
</tr>
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<td><strong>Number</strong></td>
<td>Our number system is a language for describing quantities and relationships between quantities. The value attributed to a digit depends on its place within a base 10 system. The operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used. The availability of computers and calculators has provided us with an unprecedented opportunity to explore relationships and rules in the number system.</td>
<td></td>
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<tr>
<td>• read, write and model numbers, using the base 10 system, to 1000</td>
<td>How do mathematicians write larger numbers? Why do we use base 10? causation How can we use the base 10 system to show large numbers?</td>
<td>Using numbers to 1000: Students represent 1000 in a variety of ways. Students practise using base 10 blocks to represent a certain number and practise writing numbers that have been represented by base 10 blocks. Students use base 10 materials to play trading games with an emphasis on base 10 structure and place value. Students play a calculator game that focuses on place value understanding (eg 4293: how can you change the digit 4 to a 9?)</td>
<td>Students can explain how they know what number this is. Students are given the following scenario. A new student comes to their class who has missed this unit on place value. They can explain to the new student how the base 10 system works.</td>
<td>Use place value mats/boards to extend the base 10 system, step by step, to the next place.</td>
</tr>
<tr>
<td>• count, compare and order numbers to 1000</td>
<td>Which digit is the greatest? How do we know?</td>
<td>Students use base 10 materials to play trading games with an emphasis on base 10 structure and place value.</td>
<td></td>
<td></td>
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### Number (cont.)

Students now have the opportunity to use technology to find patterns, explore relationships and develop algorithms that are practical for them. The educational experiences of students must include the use of technology.

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<th>Students will:</th>
<th>How is estimating connected to counting?</th>
<th>Counting in groups</th>
<th>How will we know what students have learned?</th>
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<tr>
<td>• estimate quantities to 1000</td>
<td></td>
<td></td>
<td>Students can explain how they estimate, how many things they think are there, how they know, and how they think they might check their estimate.</td>
</tr>
<tr>
<td>• count in 3s, 4s, 5s, and explore other numbers</td>
<td>When would it be more efficient to count by 3s, 4s…?</td>
<td>Students group manipulatives in 3s, 4s or 5s and count totals. They discuss which method was the easiest and why.</td>
<td>Find authentic contexts in your units of inquiry where estimating can be practised.</td>
</tr>
<tr>
<td>• use number patterns to learn multiplication tables: 1s, 2s, 4s, 5s, 10s</td>
<td>What number patterns do you know? When do you need to use multiplication tables?</td>
<td>Students use arrays to explore multiplication facts and the relationships between them.</td>
<td></td>
</tr>
<tr>
<td>• automatically recall basic addition and subtraction facts</td>
<td>What strategies do you use to help you learn the basic facts?</td>
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<tr>
<td></td>
<td>What are good ways to memorize the addition facts?</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>What are good ways to memorize the subtraction facts?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• model addition and subtraction equations to 1000 (with and without regrouping)</td>
<td>What equation could help us to solve this problem? What problem could this equation help us to solve? Why do we regroup? causation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Adding and subtracting to 1000</td>
<td>Students use base 10 blocks to represent addition and subtraction (to the 100s place) and, given a representation, write the equation represented by the blocks.</td>
<td>Students can calculate addition and subtraction equations, using manipulatives, from written and numerical problems.</td>
</tr>
<tr>
<td></td>
<td>Using mathematical vocabulary and symbols</td>
<td>Students explain their work to each other using new vocabulary and symbolic representation.</td>
<td>Students can correctly use mathematical vocabulary and symbols when calculating multiplication or division.</td>
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### Operations (cont.)

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<th>Students group manipulatives in 3s, 4s or 5s and count totals. They discuss which method was the easiest and why.</th>
<th>Counting on, using doubles and using tens are good strategies, to repeat the pattern and to guess the rule.</th>
<th>Students can predict, demonstrate and explain various counting patterns.</th>
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<td>Students are given manipulatives to explore patterns using 100s charts and calculators.</td>
<td>Students can input a secret counting pattern into a calculator. Others can use the constant function to repeat the pattern and to guess the rule.</td>
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<tr>
<td>Students use base 10 blocks to model addition and multiplication: times, divide product, quotient, ×, ÷.</td>
<td>Students can recognize and describe number patterns they see. They can understand to which multiplication table they are linked.</td>
<td></td>
</tr>
<tr>
<td>Students can predict, demonstrate and explain various counting patterns.</td>
<td>Research continues to indicate that “drill and kill” methods are only slightly effective and that there are better ways to practise these skills. It helps to have practised, learned and discussed strategies; students can then call upon these to help work out facts.</td>
<td></td>
</tr>
<tr>
<td>Students use their speed if they are interested. Discuss ways of improving their speed. Help them devise a way to time themselves.</td>
<td>To be useful, basic addition and subtraction facts need to be recalled automatically.</td>
<td></td>
</tr>
<tr>
<td>How best will students use mathematics in the future? How can they use mathematics to do the things they think they might check their estimate?</td>
<td>Counting on, using doubles and using tens are good strategies, although students frequently invent methods that work equally well for themselves. To increase the speed of recall, most students respond best to graphing their speed over time on a small set of similar facts.</td>
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<td>Students will:</td>
<td>Using multiple strategies</td>
<td>Sample assessments</td>
<td>Resources and comments</td>
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<td>• use and describe multiple strategies to solve addition, subtraction, multiplication and division problems</td>
<td>How can we solve this equation?</td>
<td>Students describe two strategies for solving addition and subtraction problems, and the advantages and disadvantages of each.</td>
<td>Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.</td>
<td>Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.</td>
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<td>• use and describe multiple strategies to solve addition, subtraction, multiplication and division problems</td>
<td>What mathematical operation will help us to solve this equation?</td>
<td>Students can describe two strategies for solving addition and subtraction problems, and the advantages and disadvantages of each.</td>
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<td>• use and describe multiple strategies to solve addition, subtraction, multiplication and division problems</td>
<td>Are there other ways to solve the problem?</td>
<td>Students can write a +/− equation to represent a problem situation, as well as create a context for an equation.</td>
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<td>• use and describe multiple strategies to solve addition, subtraction, multiplication and division problems</td>
<td>How accurate does the answer need to be?</td>
<td>Students are given a range of numbers. They can use several of these to write and solve problems involving a variety of mathematical operations.</td>
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<td>• use and describe multiple strategies to solve addition, subtraction, multiplication and division problems</td>
<td>How is two- and three-digit addition or subtraction the same/different when you work in your head, with paper and pencil, or with a calculator?</td>
<td>Students can write a +/− equation to represent a problem situation, as well as create a context for an equation.</td>
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<td>What equations could we use to solve this problem?</td>
<td>Students are given a range of numbers. They can use several of these to write and solve problems involving a variety of mathematical operations.</td>
<td>Students are given a range of numbers. They can use several of these to write and solve problems involving a variety of mathematical operations.</td>
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<td>Multiplication and division strategies</td>
<td>Students calculate the cost of buying the same snack/lunch every day for various periods of time.</td>
<td>Students are given a range of numbers. They can use several of these to write and solve problems involving a variety of mathematical operations.</td>
<td>Students are given a range of numbers. They can use several of these to write and solve problems involving a variety of mathematical operations.</td>
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<td>Multiplication and division strategies</td>
<td>Students calculate the total cost of a class field trip when given the cost per student.</td>
<td>Students are given a range of numbers. They can use several of these to write and solve problems involving a variety of mathematical operations.</td>
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<td>Multiplication and division strategies</td>
<td>Students determine the total cost and the number of students attending, calculate the cost per head of a class field trip.</td>
<td>Students are given a range of numbers. They can use several of these to write and solve problems involving a variety of mathematical operations.</td>
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<td>Comparing fractions</td>
<td>Can different fractions be equal?</td>
<td>Students can find and explain examples of common fractions and objects: sport courts, window panes, clock faces, egg cartons.</td>
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<td>How can we know when one fraction is greater than, smaller than or equal to another?</td>
<td>Students are asked if the following inequalities are true and can explain their thinking: ( \frac{3}{4} &gt; \frac{1}{4}, \frac{7}{4} &gt; \frac{4}{6} ).</td>
<td>Students are asked if the following inequalities are true and can explain their thinking: ( \frac{3}{4} &gt; \frac{1}{4}, \frac{7}{4} &gt; \frac{4}{6} ).</td>
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<td>Students use fraction pieces to compare fractions and to record the comparison in an equation or an inequality.</td>
<td>Students can explain what ( \frac{2}{4} ) is the same as.</td>
<td>Students can explain what ( \frac{2}{4} ) is the same as.</td>
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<td>Comparing fractions</td>
<td>Students make their own fraction strip kits using different coloured paper.</td>
<td>Given a fraction, students can build or draw a model of the fraction. Given a model, students can write the fraction using fractional notation. They can construct a whole when given a part.</td>
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<td>Students match cards using fraction words, diagrams and notation.</td>
<td>Students need practice in producing appropriate answers when using remainders. On a school trip with 25 students, and buses that carry 20 students, then a remainder could not be left behind – another bus would be required!</td>
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<td>The interpretation and meaning of remainders can cause difficulty for some students. This is especially true if calculators are being used. For example, ( 67 ÷ 4 = 16.75 ). This can also be shown as ( 16 \frac{3}{8} ) or ( 16 \frac{3}{8} ).</td>
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<td>• <strong>model addition and subtraction of fractions with the same denominator</strong></td>
<td>• How can we add and subtract fractions?</td>
<td>• Adding and subtracting fractions Students add fractions using their fraction strip kit. They find ways of recording their equations.</td>
<td>• Students can use materials and diagrams to solve simple word problems involving common fractions. Students are asked if the following equations are true: (\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}) (\frac{3}{4} - \frac{1}{4} - \frac{1}{4} = \frac{1}{4}) They can explain why or why not.</td>
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</tr>
<tr>
<td>• <strong>use mathematical vocabulary and symbols of fractions: numerator, denominator, equivalence</strong></td>
<td>• How do mathematicians write fractions?</td>
<td>• Using vocabulary and symbols of fractions Students explore fractions in a practical nature and use appropriate vocabulary to describe what they have done.</td>
<td>• Students can use correct language to describe fractions and can use symbols to read and write fractions.</td>
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<td></td>
<td>• What is a numerator?</td>
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<td></td>
<td>• What is a denominator?</td>
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<td>• What is equivalence?</td>
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<td>• Can you show fractions equivalent to 1?</td>
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<td>• What patterns do you see in equivalence to 1?</td>
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<td>• What is a reasonable estimate?</td>
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<td></td>
<td>• How do we sometimes round up or down?</td>
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<td></td>
<td>• How does place value help us with rounding?</td>
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<td></td>
<td>• Is your answer reasonable?</td>
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<td><strong>Reasonably estimate answers: rounding and approximation</strong></td>
<td><strong>Could that be right?</strong> Students estimate the answer to a problem, check their answer using a calculator or a peer and their work, and describe how they need to modify their strategy. Students read thousand numbers and practise rounding these to the nearest ten and the nearest hundred. Students suggest numbers that could round to a given number.</td>
<td>• Students are asked to evaluate the reasonableness of a variety of answers to a given problem. They can explain why or why not. Is 38 a reasonable answer to 94 - 52? Why or why not?</td>
<td>Students need many opportunities to practise estimating. It is helpful to establish a range of reasonable estimates.</td>
</tr>
<tr>
<td></td>
<td><strong>Select and explain an appropriate method for solving a problem.</strong></td>
<td><strong>Choosing a method</strong> Students are exposed to a variety of problem-solving strategies: guess and check, use simpler numbers, make a table or chart, act it out. Students share their calculation methods with the class, who can then verify or question their thinking.</td>
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</table>

PYP curriculum documents: mathematics © IBO 2003
Overall expectations

Students will have the opportunity to identify and reflect upon “big ideas” by making connections between the questions asked and the concepts that drive the inquiry. They will become aware of the relevance these concepts have to all of their learning.

Data handling: statistics and probability

Students will collect, display and interpret data in a variety of ways. They will compare data displays, including how well they communicate information. They will create and manipulate an electronic database and set up a spreadsheet using simple formulas to create graphs. They will find, describe and explain the range, mode, median and mean in a set of data, use a numerical probability scale 0-1 or 0%-100%. They will determine the theoretical probability of an event and explain why this might be different from the experimental probability.

Measurement

Students will estimate, measure, label and compare perimeter, area and volume using formal methods and standard units of measurement. They will develop procedures for finding perimeter, area and volume and recognize the relationship between them. They will use the correct tool for any measurement with accuracy. They will measure and construct angles in degrees using a protractor. They will know that the accuracy of measurement depends on the situation and the precision of the tools. They will use and construct 12-hour and 24-hour timetables and be able to determine times worldwide.

Shape and space

Students will use the mathematical vocabulary of 2-D and 3-D shapes and angles. They will classify, sort and label all types of triangle and quadrilateral. They will turn a 2-D net into a 3-D shape and vice versa. They will find and use scale and ratio to enlarge and reduce shapes. They will use the language and notation of bearing to describe position, and be able to read and plot coordinates in four quadrants.

Pattern and function

Students will understand and use the relationships between the four operations. They will model and explain number patterns and use real-life problems to create a number pattern following a rule. They will develop, explain and model simple algebraic formulas. They will model exponents as repeated multiplication, and understand and use exponents and roots as inverse functions.

Number

Students will read, write and model numbers to one million and beyond, extending the base 10 system to the millions and thousandths. They will automatically use number facts. They will read, write, model, compare and order fractions (including improper fractions and mixed numbers), decimals (to any given place), and percentages. They will interchange fractions, decimals and percentages. They will add and subtract fractions with related denominators, simplify fractions and explore fractions using a calculator. They will add and subtract decimals to the thousandths and will model multiplication and division of decimals in the context of money. They will find and use ratios; read, write and model addition and subtraction of integers; and use exponential notation. They will use and describe multiple strategies to create and solve more complex problems, reasonably estimating the answers. They will select and defend the most appropriate and efficient method.

Data handling: statistics and probability

Students will: • design a survey and systematically collect, organize and record the data in displays: pictograph, bar graph, circle graph (pie chart), line graph • create, interpret, discuss and compare data displays (pictograph, pie chart, bar/line graph) including how well they communicate information

Sample questions

• What questions do you need to ask to collect the information that will answer your inquiry? • What are the most important pieces of information we need? • How can you organize the collection of your data? • What do you know about different displays that will assist in your choice of display? • Why do you need to think about changing your survey questions? reflection

Sample activities

• Displaying survey results Opportunities for designing and implementing surveys can be found across the curriculum. Situations that come up naturally in the classroom, or as part of the units of inquiry, present opportunities for students to develop surveys and process data.

Sample assessments

• Comparing data displays Display the same data in a series of different graphical displays. Students record and share their interpretations, making judgments about the choice of display and about which display best represents the data.

Notes for teachers

Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.

Resources and comments

The collection and interpretation of data increases in complexity as students use their skill and understanding of decimal notation, fractions, percentages and measurement (scale). As well as graphs, students should use Penn and Carroll diagrams to organize and make sense of data.

It is important to remember that the chosen format should illustrate the information without bias.

* See glossary for explanation of italicized terms.
### Data handling: statistics (cont.)

**Students will:**

- find, describe and explain the range, mode, median and mean in a set of data and understand their use

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<th>Sample activities</th>
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</thead>
<tbody>
<tr>
<td><strong>Comparing data displays</strong></td>
<td>Could you display this data in a different way?</td>
<td>Students are shown familiar data in a variety of graphical displays. They discuss how the displays affect their interpretations of the data. With the same set of data, students use a computer software program to generate different graphical displays. They discuss the results.</td>
<td>Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.</td>
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<td>How would you justify your choice of display for your data?</td>
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<td>How does the representation of data change with the use of different graphical displays?</td>
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<td>Why is the information easy to read and interpret?</td>
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<td></td>
<td>What have we learned that will help us read and interpret the data? <strong>connection</strong></td>
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<td></td>
<td>Can the data be interpreted in a different way by someone else? <strong>perspective</strong></td>
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</table>

**Range, mode, median and mean**

**What mathematical skills and understandings do you need to apply to determine the range, mode, median and mean in a set of data?**

**How do you identify the range, mode, median and mean in this graphical display?**

**How can the range, mode, median and mean be altered to change the interpretation of the data?**

**Why is it important to find these points of reference for your data?**

**How is the median determined in an even set of frequencies?**

**What is the tallest? What is the difference in height between the tallest and the smallest (range)? If a new student comes into the class, how would this affect our data?**

**What is the height of each student?**

**How do you identify the mean height of 125cm; what could be the height of each student?**

**What are the mean age of all students in the class?**

**Students create a block graph using a set of interlocking blocks. They calculate the mean by taking or adding blocks to the columns so that each column becomes the same height.**

**Students measure the distance of a standing jump with paper ribbon.**

**Students are given a short specified time to repeatedly write a word. Data is collected and the range is calculated. The process is repeated for a calculation activity and the data is compared and interpreted.**

**Students order the jumps according to magnitude and then identify the range, mode, median and mean.**

**Students can demonstrate how the range is determined in a set of data.**

**Students can calculate the possible heights of the students according to the data given and can explain their working.**

**Students can explain how to find the mean in a given set of numbers.**

**Some results are not always found using whole numbers. Applying understandings of decimal notation and of rounding off can assist with these results. Practise the investigation of the mean value of data using prepared data sets that result in whole numbers.**

**Using data already collected and saved is a simple way to begin discussions of the median and to compare it with the mode. Two further extensions are to discuss why the median is sometimes identical to the mode and why sometimes it is different, and why statisticians would sometimes choose to use one method of calculating the average as opposed to the other.**
### Content: Mathematics

**Subject:** mathematics  
**Age range:** 9–12 years  
**Page 3 of 18**

#### Specific expectations

The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.

#### Sample questions

Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold.

#### Sample activities

All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen).

#### Sample assessments

Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.

#### Notes for teachers

Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives and visual aids are important for understanding for all learners.

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### Data handling: probability

**What do we want students to learn?**

Students will:
- create and manipulate an electronic database for their own purposes
- set up a spreadsheet, using simple formulas, to manipulate data and to create graphs.

**How will we know what students have learned?**

#### Probability scales

Students participate in probability activities such as tossing coins, and dice games with one or two dice.

- Students draw coloured beads with one or two dice.
- Students design and use a random spinning device.
- Students determine the possible outcomes of a chance experiment.

#### How best will students learn?**

- Using a database
  - Students use prepared databases to extract information for research purposes.
  - Students set up a class database, setting fields based on students’ physical attributes. Students extract information from the database in response to a search query.
  - Students use computer software programs to generate a spreadsheet.

- Setting up a simple spreadsheet
  - Students plan and organize a class picnic. They use a spreadsheet to calculate the cost and quantities of items needed.
  - Students investigate different spreadsheets such as the information found in charts in a travel brochure.
  - Students use computer software programs to generate a spreadsheet.

**Resources and comments**

A database is a collection of data, where the data can be displayed in many forms. The data can be changed at any time. A spreadsheet is a type of database where information is set out in a table. Using a common set of data is a good way for students to start to set up their own databases. A unit of inquiry would be an excellent source of common data for student practice.

Students will need to be taught how to use the software so that it becomes transparent. This is best done when there is an authentic purpose for learning the software.

Using a common set of data is a good way for students to start to set up their own spreadsheets.

Working cooperatively in groups will assist in the management of data entry for larger databases.

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### Probability scales

- **Will the result be the same if we repeat the experiment?**
- **Why were these the outcomes of our game?**
- **How do we make use of probability in everyday situations?**
- **Is probability predictable?**
- **What connections do we need to make between the result of the outcomes and the use of a numerical scale?**
- **How do mathematicians express the probability of an event?**
- **Why are two systems used?**

- **Probability scales**
  - Students participate in probability activities such as tossing coins, and dice games with one or two dice.
  - Students draw coloured beads from a bag.
  - Students design a game that is unfair to one or more players.
  - Students design and use a random spinning device.
  - Students determine the possible outcomes of a chance experiment.

**How is a database similar to a database? form**

- How is a spreadsheet similar to a database?
- What is the language of formulas?
- What mathematical understanding can we apply to the creation of formula?

**How is a spreadsheet similar to a database? form**

- How is a spreadsheet similar to a database?
- What is the language of formulas?
- What mathematical understanding can we apply to the creation of formula?

Data handling: probability

There are ways to find out if some outcomes are more likely than others. Probability can be expressed qualitatively by using terms such as “unlikely”, “certain” or “impossible”. It can be expressed quantitatively on a numerical scale.

The availability of computers and calculators has provided us with an unprecedented opportunity to process data and explore probability in more thoughtful, efficient and imaginative ways. The educational experiences of students must include the use of technology.
### Data handling: probability (cont.)

Students will:
- determine the theoretical probability of an event and explain why it might differ from experimental probability.

#### Theoretical versus experimental probability

Students frequently repeat experiments and compare the results. Students order events using available data to answer questions such as: Which capital city has a chance of rain tomorrow?

Given a situation, students can determine the theoretical probability of an event and can determine the experimental probability by collecting and organizing data. They can compare the two and formulate theories about what they notice.

Students understand that, as more data is gathered, experimental probability approaches theoretical probability.

Given two sets of data, students can explain why both sets can be possible results of the same experiment or survey.

#### Measurement

- select and use appropriate standard units of measurement when estimating, describing, comparing and measuring

What units should we choose?
- A range of thermometers, jugs, weighing scales etc is displayed. Students make statements about what they see, including observations of the scale. Students measure a variety of objects and record findings on a chart.

Students can take a reading from a measuring instrument. Students can find the best buy of three available sizes of the same product.

Measurements are often expressed as fractional parts.

Through their science activities, students should have the opportunity to discover the relationship between grams, millilitres and cubic centimetres in order to develop an understanding of density.

- use measuring tools, with simple scales, accurately

What do we need to consider in order to choose an appropriate instrument of measurement?
- Students select from a variety of measuring tools to complete a “body measure” of themselves. Students draw lines to given lengths, without the use of rulers, and solve the problem of measuring curved lines. Students draw and construct objects using accurate measurements.

Students can create a flow chart or other device to describe how they decide on the units and tools to use in different situations. Students can write a short passage using references to different forms of measurement (eg a shopping trip).

In real-life measuring situations, a decision about which instrument and which unit to use always precedes the actual measuring.

- understand that the accuracy of a measurement depends on the situation and the precision of the tools

How accurate is this measurement?
- Students solve problems, combining objects that must be kept at a limited weight and answer questions such as: What items can be placed in a suitcase to meet the limit of 20kg? What furniture can I buy to fit in this room?

Students can determine how accurate a measurement must be when cooking, and can defend their answers.

- How accurate can this measurement be?
- What does it depend on?

Students can use known sizes of common objects to help estimate and measure.

Measurement involves continuous quantities, so measures cannot be expressed exactly. The degree of accuracy depends on the measuring tool and the measurer.

### Resources and comments

- Students need to explore the differences between experimental probability and theoretical probability. They often have difficulty accepting the premises of theoretical probability.
- Technology gives us the possibility of rapidly replicating random events. Computer tools will toss coins, roll dice, and tabulate and graph the results.

- Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.
### Measurement (cont.)

#### Specific expectations

Students will:

- develop procedures for finding area, perimeter and volume
- determine the relationships between area, perimeter and volume

#### Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in **bold**.

#### Sample questions

- **What is area? form**
  - Does everything have area? form
  - How can we know the area of this rectangle without counting the squares?
  - How can we calculate the area of an irregular shape?

- **What is perimeter? form**
  - Are there short cuts for finding the perimeter of polygons?
  - How can we calculate the perimeter of an irregular shape?

- **What is volume? form**
  - How can we know the volume of a box without filling it with cubes?

- **Do all shapes with the same perimeter have the same area? (and vice versa)**
  - What can we know about the perimeter of a rectangle by knowing its area? connection causation
  - Why are area, perimeter and volume like they are?

- **How are the units of volume related to the units of length and area?**
  - Why do the formulas of area, perimeter and volume work? causation
  - How did you discover the formula?

- **How do we find the volume of a box without using cubes?**
  - Why is a cube used to measure volume?

#### Sample activities

- **Developing procedures for finding area, perimeter and volume**
  - Students try to make three rectangles with an area of 30cm². They are asked if there is a limited number of rectangles they can make and how they know this.
  - Students design a garden of a specific area. They cost the establishment of the garden based on its dimensions.

- **What are the relationships between area, perimeter and volume?**
  - Students use grid paper or paper tiles to investigate the possible arrays of squares for a given area.
  - Students investigate area on maps and use geoboards to model area and perimeter.

- **How do the formulas of area, perimeter and volume work? causation**
  - Students use isometric dot paper to draw a representation of cubes, and to calculate volume.

- **How best will students learn?**
  - All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen).

#### Sample assessments

- **Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.**

- **How will we know what students have learned?**
  - Students can make different rectangles with a specified area.
  - Students can systematically investigate areas of rectangles using base 10 materials to lead to the generalization of length x width.

- **Resources and comments**
  - Students can calculate volumes of 3-D representations using cubes. For example, a square-based pyramid.
  - Students can make a rectangular prism with a volume of 36cm³ and can describe how they work out the volume of a rectangular prism.

- **Notes for teachers**
  - Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.
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#### Measurement (cont.)

**What do we want students to learn?**

- **estimate, measure, label and compare, using formal methods and standard units of measurement, the dimensions of area, perimeter and volume**

What do we need to know before we can make a reasonable estimate of measurement?

- **How can we test our estimates?**

- **What do we think about when we estimate a measurement?**

- **How big is the box?**

- **What are all the possible dimensions?**

- **How can we convince someone that our measurement is correct? responsibility**

**How will we know what students have learned?**

- **Students measure the volume of quantities using centimetre cubes and calibrated volume containers.**

- **Students draw a specified polygon of specified perimeters by estimation (without using a measuring tool).**

- **Students have 48 balls with a 10cm diameter to pack into the box.**

- **Students can reflect on their estimates.**

- **Students can create a 3-D shape with a volume of 100cm³.**

- **Students can measure the volume of a box.**

### Notes for teachers

Through their science activities, students should have the opportunity to discover the relationship between grams, millilitres and cubic centimetres and should begin to develop an understanding of density.

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The computer screen can be used to display the following:

- **Using decimal notation**

  Students are given real objects and situations, in which they must make measurements, to discover that not all measurements will represent a whole number.

  Students use a calculator to investigate counting in decimal sequences.

  Students solve the following problem: In a race with five competitors; the winner had a time of 12.5 seconds and the last competitor had a time of 15.2 seconds, what could the times of the other competitors be?

  Students can describe a situation where they would need, or want, to record the measure of an object to the tenths or hundredths place. They can explain why using decimal notation helps them to understand and describe the measurement.

<table>
<thead>
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<tr>
<td>Students create rotational patterns from a stencil and fix them with a single pivot point. They rotate the stencil to a named angle and then trace.</td>
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<thead>
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<tr>
<td>Students investigate the use of angles in geometric artworks. Students draw patterns within circles.</td>
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<th>Understanding an angle is a measure of rotation</th>
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<td>Students draw rotational patterns from a stencil and fix them with a single pivot point. They rotate the stencil to a named angle and then trace.</td>
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<td>Students investigate the use of angles in geometric artworks. Students draw patterns within circles.</td>
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### Measurement (cont.)

**Students will:**

- measure and construct angles in degrees using a protractor
- use and construct timetables (12-hour and 24-hour) and time lines

#### Measuring and constructing angles

**Sample questions**

- What language do we use to describe angles?
- How can we identify the different angles?
- What are the relationships between the different angles?
- Where are angles to be found in the immediate environment?
- Why are angles a useful form of measurement?
- How do we use a protractor to create or measure a given angle?
- How can we calculate angles without the use of a protractor?
- How are angles connected to a circle?
- What are the relationships between angles and geometric shapes?
- How does the measurement of angles relate to shapes?
- How does the measurement of shapes relate to symmetry?

**Sample activities**

- Students fold squares of paper. They describe the shapes and angles created using the specific vocabulary of angles.
- Students use two straws, joined at an end pivot point, to create and draw angles. They order the size of angles and discuss.
- Students draw a chart of named angles to see the unique features of each angle and also to see the relationships between angles.
- Students measure angles in a real-life context. *Model use of the protractor.*
- Students investigate architectural drawings, art forms, bridges and buildings. Students draw a pathway in a maze following a set of instructions that uses angles.

**Sample assessments**

- Students can identify angles in their immediate environment, discuss the possible measurements of these angles and justify their decisions.
- Students can use a protractor to measure angles appropriately.
- Students can draw a rectangle and check to see if it is correct.

#### Using time lines and timetables

**Sample questions**

- What is a 24-hour clock? *form*
- Why do we use 12-hour and 24-hour times?
- How do 12-hour and 24-hour times relate? *change, connection*
- How do you manage your time over extended periods? *responsibility*
- How can you plan a journey using a timetable? *reflection*
- Why do you need to understand to make sure you will be on time at a destination? *responsibility*
- Why is it important to measure time accurately?
- What time would be one minute before 15.00 hours?

**Sample activities**

- Students brainstorm situations that use 24-hour time.
- Students synchronize times on a 12-hour and a 24-hour clock and discuss.
- Students use authentic rail, train and bus timetables to plan a journey within an authentic context.
- Students play card games, matching cards with 12-hour and 24-hour times.
- Students investigate the history and use of 24-hour time.
- Students calculate the time elapsed between two given times.
- Students solve problems using authentic timetables (eg a TV programme).
- Students use analogue clocks to count on or count back to calculate elapsed time.

**Sample assessments**

- Students can complete a class timetable on a blank form to a set of given criteria.
- Students can assist with the organization of a camp timetable.
- Students can create a time line from midnight to midnight.
- Students can share the strategies used to convert 12-hour time to 24-hour time and vice versa.
- Students can answer questions such as: The time is now 25 minutes to 4; how many different ways can this be written? A swimming lesson takes 1 hour and 10 minutes; what could be the start and finish times of the lesson?
### Subject: Mathematics  
**Age range:** 9–12 years  
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<table>
<thead>
<tr>
<th>Content</th>
<th>Specific expectations</th>
<th>Sample questions</th>
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<th>Notes for teachers</th>
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<td>Measurement (cont.)</td>
<td>The specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.</td>
<td>Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold.</td>
<td>All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen).</td>
<td>Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.</td>
<td>Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.</td>
</tr>
<tr>
<td><strong>Shape and space</strong></td>
<td>Students need to understand the interrelationships of shapes, and the effects of changes to shape, in order to understand, appreciate, interpret and modify our two-dimensional and three-dimensional world.</td>
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<td>This aspect of measurement is best taught in real-life contexts.</td>
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<td>Students will:</td>
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| | • determine times worldwide. | What are time zones? Why are there time zones? causation What time is it now in your home country? connection | Times around the world  
Students use atlases and globes to investigate time zones. Students participate in video conferencing link-ups with other schools. Students interpret flight schedules. | When given a time in one country, students can calculate the time in another country. Students can investigate the occurrence of flying to a country on one day and landing there on the same day. | |
| | • use the geometric vocabulary of 2-D and 3-D shapes: parallel, edge, vertex | What language is used to describe the features of shapes? What features do 2-D shapes have in common? What features do 3-D shapes have in common? What is an edge? What is a vertex? What is a face? How are the features of 3-D shapes related to one another? | Using the vocabulary of 2-D and 3-D shapes  
Shape Show and Tell  
Students select a 3-D shape, list all the possible properties of the shape and share this information with others. Students create shapes using straws. Students use geoblocks and similar materials as resources for identifying the critical attributes of the shapes. | From a list of 3-D shapes students can list what real-life shapes they could be (eg a sphere and a ball). Students can write a procedural text explaining how to draw a particular shape. Students can use the terms vertex, edge and face to describe the attributes of 3-D shapes found in their immediate environment. | |
| | • classify, sort and label all types of triangles and quadrilaterals: scalene, isosceles, equilateral, right-angled, rhombus, trapezium, parallelogram, kite, square, rectangle | What do all triangles have in common? function What do all quadrilaterals have in common? function Can you change one quadrilateral into another? change How could you describe a quadrilateral to someone who has never seen one? reflection How does the name relate to the shape? How does the vocabulary used to describe a shape relate to the features of that shape? | Sorting triangles  
Students explore the features of these shapes to develop their own conceptual framework. Students create a class poster of shapes and their descriptions. Students match a series of shapes to the text descriptions. Students draw a series of four-sided shapes. They discuss the similarities and differences. They do the same for triangles. | Students can identify and sort a wide selection of special triangles and quadrilaterals and can categorize them in at least two ways. | |
| | • understand and use the vocabulary of types of angle: obtuse, acute, straight, reflex | What words do mathematicians use to describe the different types of angles? Why are there distinctions among these types of angles? How are these angles connected to one another? Where can we see examples of these shapes in the immediate environment? | Using the vocabulary of angles  
Students begin to identify and classify different types of angles through measurement activities and exploring shapes. | Students can identify the types of angles in their immediate environment, using the correct vocabulary, and can defend their observations. | Students need to understand this specialized vocabulary, and the relationship between the features of a shape and its name. |

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**Notes for teachers:**  
- Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.  
- This aspect of measurement is best taught in real-life contexts.
Subject: mathematics  
Age range: 9–12 years  
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<td>That the specific expectations must be carefully addressed to ensure learners have FULLY understood a mathematical concept before moving on. These expectations represent the desired understanding to be achieved by the end of this age range.</td>
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<td>• understand and use geometric vocabulary for circles: diameter, radius, circumference</td>
<td><strong>Specific expectations</strong></td>
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<td><strong>Using the vocabulary of circles</strong></td>
<td>Students investigate drawing circles using a pencil or chalk tied to a piece of string. This will lead to using a pair of compasses to draw circles.</td>
<td>Students can compare and contrast two circles using the vocabulary of circles. Students can draw a circle with a diameter of 1.4cm or a radius of 8cm.</td>
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<td></td>
<td>How can you make or model a circle?</td>
<td>How are all circles the same?</td>
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<td>What makes a circle unique?</td>
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<tr>
<td>• use a pair of compasses</td>
<td><strong>Using a pair of compasses</strong></td>
<td>Students follow a set of instructions to create a pattern that is based on a line or a shape.</td>
<td>Students can use a pair of compasses to investigate how a particular shape may have been constructed. Students can give instructions on how to produce a particular shape.</td>
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<td></td>
<td>How is a pair of compasses related to mathematics?</td>
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<td>What can be drawn with a pair of compasses?</td>
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<td>• understand and use the vocabulary of lines, rays and segments: parallel, perpendicular</td>
<td><strong>Using the vocabulary of lines</strong></td>
<td>Students draw sets of two lines and discuss them. Students find parallel lines in the classroom. They use a spirit level to identify horizontal and vertical lines in the classroom and then the parallel lines. Students identify sets of parallel lines in given quadrilaterals. Students identify lines of symmetry in shapes.</td>
<td>Students can construct a line that is perpendicular or parallel to another. Students can draw a straight and a curved set of parallel lines and can explain how they constructed them. Students can identify and mark all parallel lines as seen in a geometrical drawing. They can make a design using only parallel lines. They can draw a shape of an object that has 2, 3 or 4 lines of symmetry.</td>
<td></td>
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<tr>
<td></td>
<td>What are lines, rays and segments?</td>
<td>How are they related to each other?</td>
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<td></td>
<td>How can we describe the relationship between lines?</td>
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<td>When are two lines parallel?</td>
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<td>What is a feature of parallel lines?</td>
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<td>Where do you find parallel lines?</td>
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<td>How do the names of some shapes reflect the lines used to draw the shape?</td>
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<td>• describe, classify and model 3-D shapes</td>
<td><strong>Exploring the features of 3-D shapes</strong></td>
<td>Students explore the features of shapes to develop their own conceptual framework. Students create a class poster of shapes and their descriptions. They match a series of shapes to the text descriptions. Students use perspective to draw 3-D shapes.</td>
<td>Students can sort a cube, sphere, triangular prism and square-based pyramid into two groups and justify the reasons for placing the shapes in these groups. Given a photograph of architectural structures, students can deconstruct the buildings into their constituent shapes using appropriate vocabulary.</td>
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<td></td>
<td>Why are 3-D shapes named as they are?</td>
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<td>What are the similarities and differences in 3-D shapes?</td>
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<td>What clues do the names of shapes give us to recognize and draw the shape?</td>
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<td>Where does the term polyhedra originate? Can you sketch 3-D shapes as 2-D shapes?</td>
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<td></td>
<td>What patterns do you notice in the cross-sections of these 3-D shapes?</td>
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<td>Students cut cross-sections of clay cones, cylinders, pyramids and prisms. They measure their dimensions and look for patterns. Students investigate how artists draw 3-D shapes. Students can predict which cross-sections come from which 3-D shapes and can defend their prediction. Students can draw a robot using 3-D shapes. Students can locate 3-D/2-D shapes within artwork. Students can investigate the patterns that occur between shapes when counting edges, faces and vertices (Euler).</td>
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<td>The purpose of this study is for students to understand specialized vocabulary: polyhedra, spheres, cylinders, cones, prisms and pyramids.</td>
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<td>This is a good opportunity to conduct an investigation whose outcomes will lay a foundation for higher level algebra and geometry.</td>
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<td>Leonhard Euler was a mathematician who was the first to discover these relationships.</td>
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## Shape and space (cont.)

**The regions, paths and boundaries of natural space can be described by shape.**

**Students will:**

- turn a 2-D net into a 3-D shape and vice versa
- find and use scale (ratios) to enlarge and reduce shapes
- use the language and notation of bearing to describe position

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### Turning 2-D shapes into 3-D shapes and vice versa

- What is a net?
- How are 2-D shapes related to 3-D shapes?
- How is a net created?
- What do these shapes have in common?

#### Using scale

- What is scale?
- What is a ratio?
- What do we need to understand to enlarge or reduce a shape?
- How is scale used in real life?
- How is a ratio interpreted?
- What mathematical processes are used to calculate and interpret scale?
- Why do plans and maps use scale?

**Students will:**

- Students use interlocking plastic shapes to create 3-D shapes.
- Students are given sets of congruent shapes, made from cardboard, to tape together.
- Students use straws, sticks and modelling clay to make skeleton shapes.
- Students make a city skyline from various nets.
- Students create a container/box for a toy.
- Students use prepared grid paper to assist in making nets.

**Students can investigate the following problem: A set of six squares was joined together. They could not be folded to make a 3-D shape. What might the pattern of squares look like?**

**Students can design a net for a box that has a volume of 150 cubic units.**

**Students can estimate the scale of selected objects (toys).**

**Students can draw a reduced image of an object (chair) and can explain how they did this and what scale means.**

**Students can interpret maps using scale.**

**Students can use rectangular grids to investigate distortions in images.**

**Students can solve a problem such as: “On my biking holiday I want to travel between 1000km and 1200km. Which towns can I visit on my journey?”**

**Students should see the relationship between angles and shapes.**

**Students should see the relationships between a half turn and 180 degrees.**

**Students will become aware of the conventions of north and south being named first in a direction eg north-east.**

**Navigational systems are based on bearings.**

---

### Finding a hearing

- How are angles used to describe a bearing?
- How can we describe the position of something?
- What is the language used to describe position?
- What is the notation used to describe position?

**Students investigate afloats and maps, and do mapping exercises.**

**Students give directions to other students who are blindfolded.**

**Students investigate the different ways in which accurate directions are given.**

**Students follow a set of written instructions, using three digit notation of bearings, to move themselves or objects from one position to another.**

**Students can describe a journey undertaken by an explorer such as Captain Cook.**

**Students can design their own grid drawing with accompanying instructions.**

**Students can plan and undertake an orienteering course.**

**Students can write a set of three directions from a designated starting point to a designated end point using bearings. They can follow each other’s directions.**

---

### Resources and comments

- Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners.

- Students require many varied activities in handling 3-D shapes so that they see how the properties of shapes relate to the name given to those shapes.

- Students require an understanding of the number and shapes of faces of 3-D shapes before they are able to construct nets.

- The idea of scale is very important in geography. This mathematical concept is best practised through a unit of inquiry.
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<td>• read and plot coordinates in four quadrants.</td>
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<td>Students can identify a pattern around the grid.</td>
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<td>Students can identify and explain positions of given numbers on a grid.</td>
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<td>Students can plot a point on a grid according to certain specifications.</td>
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<td>Students can identify positions on a grid by making a series of related points.</td>
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<td>Students can identify positions on a grid using Cartesian coordinates.</td>
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<td>Students can list coordinates for given positions.</td>
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<td>Pattern and function</td>
<td>To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called functions. This builds a foundation for the later study of algebra.</td>
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<td>• understand and use the relationship between multiplication and addition</td>
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<td>How are addition and multiplication connected?</td>
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<td>How is multiplication repeated addition?</td>
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<td>The relationship between multiplication and addition</td>
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<td>Using calculators, students explore the results of repeated addition against multiplication of numbers.</td>
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<td>Students can use and explain the relationship between addition and multiplication.</td>
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<td>• understand and use the relationship between multiplication and division (inverse function)</td>
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<td>How are multiplication and division related?</td>
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<td>Using tiles, students create rectangular arrays that have equal area but different dimensions.</td>
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<td>Students can calculate the possible dimensions of a garden when its area is 24 square units.</td>
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<td>The relationship between division and subtraction</td>
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<td>Using manipulatives and calculators, students explore this relationship.</td>
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<td>How are division and subtraction related?</td>
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<td>• model and explain number patterns</td>
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<td>How are numbers arranged to create a pattern?</td>
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<td>What is the relationship between the numbers?</td>
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<td>How can you explain what will be the next number in this pattern?</td>
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<td>How can a number relate to a function?</td>
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<td></td>
<td>Where can number patterns be found in real-life situations?</td>
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<td></td>
<td>Explaining number patterns</td>
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<td></td>
<td>Students investigate the patterns in multiplication tables.</td>
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<td>Students use a circle, with numerals 0–9 placed evenly around the circumference, to create shapes by drawing to the number that forms the last digit of the product: a 10 point star is created when investigating 3s.</td>
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<td>Students identify and explain products from a multiplication table on a 100s chart.</td>
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<td>Students draw “Input Output Machines”, each with an identified function, and answer questions such as: What will be the output if the input is 4?</td>
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<td>Students investigate Fibonacci sequences.</td>
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<td>Students can continue a pattern by using plastic counters.</td>
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<td>Students can continue patterns by making a series of congruent shapes on grid paper.</td>
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<td>Students can create a pattern from a given starting point.</td>
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<td>Students can identify a pattern and explain the pattern or rule in a series of related numbers:</td>
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<td>39, 93, 152, 285</td>
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<td>Students can follow and determine a rule.</td>
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<td></td>
<td>Students can use a calculator to complete functions.</td>
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### Content | Specific expectations | Sample questions | Sample activities | Sample assessments | Notes for teachers
--- | --- | --- | --- | --- | ---
**Pattern and function (cont.)** | Students will: | | | | |
• use real-life problems to create a number pattern, following a rule | How can functions help us to describe real-life problems? | Creating patterns that follow a rule | Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways. | Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners. |
• develop, explain and model simple algebraic formulas in more complex equations: \( x + 1 = y \), where \( y \) is any even whole number | What symbols do mathematicians use to represent numbers that are unknown? | Cracking codes | Algebra is a mathematical language using numbers and symbols to express relationships. When the same relationship works with any number, algebra uses letters to represent the generalization. Letters can be used to represent the quantity. Students will need to understand that an equation has a balance: \( 2 + x = 5 \). Students progress to understanding that some algebraic formulas have a range of solutions. |
• model exponents as repeated multiplication | How is an exponent? | Exponents as repeated multiplication | Students can use concrete materials to model and explain square numbers. | A prerequisite to the use of exponents is the understanding of multiplication and the notion of factor. An exponent tells us how many times the number is a factor. |
understand and use exponents and roots as inverse functions: \( 9^7, \sqrt{81} \). | How are multiplication and division connected? | Using exponents and roots | Students can explain how \( x \) squared is related to the square root of \( x \), and how they know this. | |
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### Subject: mathematics  
**Age range:** 9–12 years  
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<td><strong>Number</strong></td>
<td>Our number system is a language for describing quantities and relationships between quantities. The value attributed to a digit depends on its place within a base 10 system. The operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used. The availability of computers and calculators has provided us with an unprecedented opportunity to explore relationships and rules in the number system. Students now have the opportunity to use technology to find patterns, explore relationships and develop algorithms that are meaningful to them. The educational experiences of students must include the use of technology.</td>
<td>Students will:</td>
<td>Using numbers to a million</td>
<td>Using two sets of 0–9 cards. Draw out seven cards. Make the highest and lowest number. Ask six students to write their number on a card and see whether they can place these in order from the largest to the smallest. Allow students to discuss the strategies they used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>How does the base 10 system extend to millions and beyond?</td>
<td>Students are presented with the following scenario: a new student joins their class and has missed this unit on place value. Can you explain to them how the base 10 system works.</td>
<td>Assessments should be directed to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.</td>
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<td>What is place value?</td>
<td>Students present a counting system to the class.</td>
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<td>What does place value relate to how we read large numbers?</td>
<td>Students investigate the occurrence of a decimal point in a number.</td>
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<td>What is a decimal?</td>
<td>Discuss with students what a decimal place represents. Money and measurement are good examples for discussion of decimal notation.</td>
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<td>How is a decimal represented?</td>
<td>Students use base 10 material. The 1000 block now represents one unit.</td>
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<td>How are decimals related to whole numbers?</td>
<td>Model numbers using graph paper.</td>
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<td>How are decimals connected to the number 1?</td>
<td>Students key a number into a calculator. They make the number ten times larger but predict what will happen before they key in ÷ 10.</td>
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<td>How does a value change in a place value system?</td>
<td>Students can explain how, in our place value system, the places to the right of the decimal point resemble and differ from the places to the left. They can explain what a decimal fraction is.</td>
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<td></td>
<td>How are the different places in a place value system related to one another? connection</td>
<td>Students can explain how the base 10 system works to another student and can demonstrate this using manipulatives.</td>
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<td>What is place value?</td>
<td>Students know which digit has the greatest value in the number 22 022. They can explain how they know this.</td>
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<td>• automatically recall and use basic number facts</td>
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<td>The introduction of a remainder must only follow after students have a strong understanding of the division algorithm. Later, decimal notation can be introduced into the answer.</td>
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<td>• create and solve multiple digit multiplication and division problems</td>
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<td>Related denominators are those such as halves, quarters and eighths. Through equivalence, students can substitute so that all denominators are the same. A fraction is part of a whole. Whole numbers can also be expressed in the same way. Explore fractions at <a href="http://www.math.rice.edu">www.math.rice.edu</a>.</td>
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<td>• read, write and model addition and subtraction of fractions with related denominators</td>
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<td>Sample questions</td>
<td>Number facts</td>
<td>Students are exposed to a variety of activities and strategies to enable them to construct meaning about basic number facts. They explore several methods and discuss these with other students.</td>
<td>Students can match cards with an example of an algorithm and the worded strategy.</td>
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<td>Students can explain the steps to multiplying 45 × 23. Students can make up word problems such as: Use 9, 8, 7 and use ×. How many different answers can you get? Students can match cards with an example of an algorithm and the worded strategy. Students can explain whether the following equations are true or false: 5⁄6 + 3⁄4 + 5⁄4 = 1, 1 = 5⁄6 − 1⁄4 × ½ = 0.</td>
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<td>Multiplication and division problems</td>
<td>Students investigate multiplication by 10 and develop a rule. Units become tens and there are no ones. Students combine understandings of multiplying by 10 and multiplying by a single digit. They check answers by rounding off and approximating an answer.</td>
<td>Students can explain the steps in multiplying 45 × 23. Students can make up word problems. Students can solve problems such as: Use 9, 8, 7 and use ×. How many different answers can you get?</td>
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<td>Addition and subtraction of fractions</td>
<td>Students are provided with various materials to explore fractions. They use these to: • make models of fractions • create the same fractions from different materials • identify how many parts make a whole • compare fractions • discover equivalent fractions. Students use fraction strips and fraction dice to add and subtract fractions.</td>
<td>Students can explain whether the following equations are true or false: 5⁄6 + 3⁄4 + 5⁄4 = 1, 1 = 5⁄6 − 1⁄4 × ½ = 0.</td>
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<td>• read, write and model improper fractions and mixed numbers</td>
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<td>• compare and order fractions</td>
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<td>• model equivalency of fractions: ( \frac{2}{4} = \frac{1}{2} )</td>
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<td>• simplify fractions</td>
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<td>• use the mathematical vocabulary of fractions: improper, mixed numbers</td>
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### Improper and mixed fractions
- Colour a Fraction game: Students roll a fraction dice and colour the parts of a shape. Extend this game so that answer sheets may have more than one line showing halves. It may be possible to colour in 6 halves.
- Pizza game: Students roll a dice to make up pizzas from fractional slices.
- Students fill labelled bottles by using \( \frac{1}{2} \) cup measures.
- Students use fractional number lines to see the relationship between improper fractions and mixed numbers.

### Comparing and ordering fractions
- Students use number lines to show the order of fractional parts.
- Students use a number line with marked fractional intervals to count in fractional steps.
- Students use fractional number lines to see the relationship between improper fractions and mixed numbers.

### Equivalent of fractions
- Students use fraction kits to model fractions. They can demonstrate making of “wholes” using fractional kits and explain using mathematical language.
- Students can use Cuisenaire rods to show the relationships and understandings of improper fractions.

### Simplifying fractions
- Students can demonstrate why two fractions are equivalent.
- Record a series of equivalent fraction pairs. Students can explain why each pair is the same.
- Two fractions added together give a total of \( \frac{1}{2} \). Students can explain what these two fractions might be.

### Using vocabulary of fractions
- Students can use the correct mathematical language and notation to describe fractions.
- They can group fractions and describe them accurately.

## Notes for teachers

- Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways.

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<td><strong>read, write and model multiplication and division of decimals (with reference to money)</strong></td>
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<td><strong>round decimals to a given place or whole number</strong></td>
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<td><strong>read, write and model percentages</strong></td>
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| Number (cont.) | | | | |
| Students will: | | | | |
| * interchange fractions, percentages and decimals | | | | |
| Questions that address the key concepts (Fig 5 Making the PYP happen) challenge learners and promote genuine understanding. All of the sample questions can be linked to a key concept. Some examples are noted below in bold. | All activities encompass some, or many, of the specific expectations and transdisciplinary skills (Fig 14 Making the PYP happen). | Assessments should be directly related to the specific expectations. Students should be given the opportunity to demonstrate their understanding in a variety of ways. | Teachers should find ways to ensure EAL learners understand tasks and expectations. Manipulatives facilitate understanding for all learners. |

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<td>How are percentages, fractions and decimals related?</td>
<td>Interchanging fractions, percentages and decimals</td>
<td>Students can explain that percentages are another way of representing a fractional part, using 100 as the denominator. Create TRIOMINOES with a fraction, decimal or percentage fraction written along one of the three sides. Students can play a game to match the relationships.</td>
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<td>Why can there be an interchange between these?</td>
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<td>How can we work out how much we are saving when buying sale articles?</td>
<td>Students investigate percentage of a metre ruler (eg 75cm is 75%). Students write a short story of 100 words. They look for the percentage of certain words used (eg and, of). They show percentages on a 100s chart.</td>
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| * find and use ratios | | | | |
| Where are ratios found in everyday situations? | Using ratios | Students can use ratios in drawing plans to scale. They can correctly increase or decrease ingredients in cooking recipes according to the ratio required. | A ratio expresses the relationship between two numbers or quantities. |
| How is a ratio used to express a relationship? | Students mix cordial to find out what ratio of water and cordial is needed. Mapping work. Cooking experiences. | |

| * read, write and model integers | | | | |
| In which mathematical situations are negative numbers found? | Using integers (negative numbers) | Students record the temperature of water as it is cooled and heated. Students investigate the terms “in the black and in the red” in relation to banking (withdrawal and depositing money). | The notation of negative numbers must clearly show that negative 1 is not the same as –1 (subtract 1). |
| How does a negative number connect to addition and subtraction? | | | |
| How can a negative number be shown on a number line or scale? | |

| * read, write and model addition and subtraction of integers (negative numbers) | | | | |
| What happens when a negative number and positive number are added? Does that always happen? | Addition and subtraction of integers | Students can calculate the daily temperature rise/fall in various parts of the world. | Addition and subtraction of negative numbers can cause confusion. Number lines and visual aids are helpful in overcoming this. |

<p>| * read, write and model exponential notation | | | | |
| How are exponents related to multiplication and division? Why are exponents used in mathematics? | Using exponential notation | Students can describe exponents as repeated notation and can use exponential notation correctly. | A sound understanding of multiplication, factors and large numbers is required before working with exponents. |
| | Students create a table to show factors and the product. the number of times the digit is a factor and the shortened recording: | | |
| | $3 \times 3 = 9$ | $2 \times 2 \times 2 \times 2 \times 2 = 32$ | |
| | $3 \times 3 \times 3 = 27$ | $3 \times 3 \times 3 \times 3 = 81$ | |
| | They create arrays in grids or using blocks. | | |</p>
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<td>• select and defend the most appropriate and efficient method of solving a problem: mental estimation, mental arithmetic, pencil and paper algorithm, calculator.</td>
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<td>Number (cont.)</td>
<td>Choosing the best method to solve problems</td>
<td></td>
<td>From a variety of possible strategies, students can select the most efficient method and explain how they reached their conclusion. Students can estimate an answer to a problem, check it against a calculator and then explain why or why not they should/should not modify their strategy.</td>
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<td>Students are given a range of written and numerical problems to solve. They have the opportunity to change their methods, discuss their changes and to explain their choices.</td>
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<td>How is mathematics a language?</td>
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<td>How do we interpret mathematical language?</td>
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<td>What cues are in language to help us select the operation needed to solve a problem?</td>
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<td>How do we know when to use a particular process?</td>
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<td>What words are signposts to choosing an operation to solve a problem?</td>
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<td>When do we use estimation in mathematics?</td>
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